CASCADE WIND TUNNEL FOR TRANSONIC COMPRESSOR BLADING STUDIES

Willard Joe Demo

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

CASCADE WIND TUNNEL FOR
TRANSONIC COMPRESSOR BLADING STUDIES

by

Willard Joe Demo, Jr.

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machine drawings for the important components of the facility are included.

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CASCADE WIND TUNNEL FOR TRANSONIC COMPRESSOR BLADING STUDIES

by

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NOMENCLATURE

English Letter Symbols

A	-	Area, in^2
AR	-	Aspect ratio
B	-	Nozzle width, in
C_L	-	Lift coefficient
℄	-	Center-line notation
c	-	Blade chord, in
c_p	-	Specific heat at constant pressure, for air = .24 Btu/lb^oR
f	-	Fraction of blade passage between end blade and nozzle wall
H	-	Nozzle height, in
i	-	Incidence angle, degrees
k	-	Ratio of specific heats, for air = 1.4
L	-	Lift, lbf
M	-	Mach number
$\dot{m}$	-	Mass flow rate
N	-	Number of blade passages
P	-	Pressure, lbf/in^2
q	-	Dynamic pressure, lbf/in^2
R_o	-	Radius of settling chamber, in
r	-	Radius, in
s	-	Blade spacing, in
V	-	Velocity, ft/sec
W	-	Velocity relative to rotor blade, ft/sec
X	-	Dimensionless velocity, $V/\sqrt{2c_pT_t}$
x	-	Distance along nozzle, in
y	-	Height from nozzle centerline to contoured wall, in
z	-	Scale factor

Greek Letter Symbols

β - Relative flow angle, degrees

γ - Stagger angle, degrees

ϕ - Camber angle, degrees

ρ - Density, lb/ft^3

Δ - Finite difference

δ^* - Boundary layer displacement thickness, in

ν - Kinematic viscosity, ft^2/sec

σ - Solidity, ratio of c/s

δ - Deviation angle, degrees

μ - Mach angle

Subscripts

0 - Settling chamber

1 - Section at cascade inlet or supersonic nozzle outlet

2 - Section at cascade exit

w - Nozzle wall conditions

L - Lift coefficient

t - Total or stagnation properties

∞ - Reference condition for cascade

Superscripts

$\bar{}$ - Condition at outer edge of boundary layer

* - Denotes conditions at nozzle throat or sonic flow

I. INTRODUCTION

The purpose of the work reported here was to design and construct at low cost a small transonic cascade wind tunnel to be used in the Transonic Compressor Research Program at the Naval Postgraduate School (NPS), Monterey, California.

If the stream surface, through either a rotor or a stator, of a compressor is unwrapped and laid out in a plane, a two-dimensional cascade is generated. The cascade, with an infinite number of blades, is considered to model the flow along the particular stream surface through the compressor. The characteristic feature of a cascade is that the flow repeats itself identically over each blade pitch, a condition referred to as periodicity. Cascades are generally used to gain better understanding of the complex flow in axial turbo machines and to gather data on the losses, pressure increase and turning produced by given blade sections.

In order to model the flow through a rotor, it is the flow relative to the blading which must be established in the stationary cascade. The picture is then as shown in Figure 1. The blade shape, spacing (s), chord (c) and stagger angle (γ) are those of the compressor blading on the stream surface of interest. The approaching flow velocity is that of the flow relative to the moving rotor (W_1), and the outlet velocity, which can be determined

experimentally in the cascade, corresponds to the relative flow velocity leaving the rotor (W_2).

In an effort to develop smaller and more compact axial compressors, rotational speeds have been increased to input more energy per stage. Through-flow velocities have also been increased to increase the mass flow per unit frontal area through the compressor. As a result, the inlet velocity relative to the rotor blades can become supersonic and, depending on the particular blade configuration, the blade passage and relative exit velocities can be either supersonic or subsonic. Flows through blading, which contain regions of both supersonic and subsonic flow, are termed transonic, and various types of transonic cascades have been described. (Ref. 1) The "shock-in-rotor" type of transonic flow pattern is the one of interest in the present work.

A Transonic Compressor Research Program at the Naval Postgraduate School was begun in 1969. The overall goal of the program was to understand the flow phenomena in advanced compressors in order to propose and to verify methods of designing axial compressors for improved performance. The main experimental tool in this program is a single stage transonic compressor designed by Dr. M. H. Vavra. An outline of the compressor is shown in Figure 2. The rotor blades in the machine have profiles which consist of a flat surface on the pressure side and a simple circular arc on the suction side, joined by small radii at the

leading and trailing edges. The rotor was designed to operate with a relative Mach number of 1.5. The flow pattern is expected to be of the "shock-in-rotor" type. Tests of the compressor have established performance map data (Ref. 2), rotor blade-element performance (Refs. 3 and 4) and case wall pressure maps at the rotor tip (Refs. 5 and 6).

Measurements of the flow leaving individual rotor blade passages also have been made (Refs. 6 and 7). Currently, compressor test speeds have been increased to 21300 RPM which is 70% of the design speed. At this speed, the velocity relative to the rotor blades is just sonic. Figure 3 shows a case wall pressure map obtained at 70% design speed using newly developed high speed data acquisition techniques (Ref. 5). The map describes the pressure pattern which rotates with the rotor tip. It was of interest to compare these measurements, and similar measurements at higher speeds, with either measurements from or analysis of an equivalent two-dimensional cascade model. Since an analysis was not then available for the transonic cascade, a two-dimensional cascade, which modelled the stream surface at a chosen relative Mach number of 1.4, was designed and constructed.

The present document details the design of the cascade. In Section II, the important considerations in the design are outlined. In Section III, a description is given of the general layout, with mention of specific items which required the most design effort. Detailed design calculations are given in Section IV. Two generally useful design procedures

were developed in the course of the design, and these are given in the appendices. First, Appendix A describes the design of a smooth, round-to-rectuangular transition section which was required upstream of the two-dimensional nozzle. Then, the design of the nozzle is described in Appendix B. Inviscid contours used in the supersonic section were those obtained from the Naval Surface Weapons Center (NSWC), White Oak, Maryland. However, a program was written for the HP-9830A calculator to design supersonic nozzle contours; the program and a comparison with the NSWC design are included in Appendix B. Appendix C contains a general procedure for calculating the pressure drop in a sudden expansion process. Machine drawings for the components of the cascade are included in Appendix D.

II. DESIGN CONSIDERATIONS

Important factors to be considered in the design fall into two categories. First are the limitations due to the available facilities, siting and equipment. Second are considerations specific to the design of the cascade itself. Figure 4 illustrates the arrangement of the resulting cascade design and will be referred to in the following discussion.

A. FACILITY LIMITATIONS

In order to use the air supply system available in building 230 at NPS, the cascade was designed to operate as an intermittent blow-down wind tunnel. Test section flow area was then limited by the available mass of stored air and the minimum desired run time. For a given flow area, the number of blades in the cascade was then determined by the magnitudes of the stagger angle, aspect ratio and solidity. Stagger angle was given by the compressor geometry. The aspect ratio could be chosen. Higher aspect ratios would minimize the effect of the side wall boundary layer on the two-dimensionality of the flow. However, because the blade profile in the compressor was very thin, structural considerations were decisive, and an aspect ratio of one was selected. Solidity was also given by the

compressor geometry. But, as solidity is the ratio of blade chord to blade spacing, each could be scaled by the same factor.

A runtime of two minutes was selected as the minimum required to acquire data. Tests were conducted with the existing air system using simple rectangular nozzles, of varying sizes, operating as a free jet. The nozzle area giving a two minute run time was selected for the throat area for the supersonic nozzle. The nozzle exit area was calculated for a Mach number of 1.4, and the stagger angle, aspect ratio and solidity selected earlier, determined the geometry shown in Figure 4. The cascade consisted of 6 blades, with a fractional passage width bypassed at each end blade. The end blades are half blades which present the correct stream boundary for the flow through five blade passages.

B. CASCADE DESIGN CONSIDERATIONS

Uniform supersonic flow upstream, generated by a properly contoured Laval nozzle, satisfies the requirement for periodic inlet flow to the cascade. However, as shown in Figure 4, the shock waves generated by the blade leading edges will be reflected from a solid upper nozzle wall and modify the flow into the upper blade passages. Therefore, a slotted section, which is vented to an exhaust line, will be provided to minimize the strength of the reflected disturbances.

Periodic exit flow is no less important and is reported to be more difficult to achieve (Ref. 8). As shown in Figure 4, the present design removes a portion of the flow from the top and bottom walls via separate exhausts, and the main flow is exhausted into a dump diffuser. The main flow is throttled downstream by a valve which provides the necessary back pressure for the cascade.

The boundary layers on the top and bottom walls are prevented from entering the blading since they are removed through the by-pass exhaust ducts. The side wall boundary layers do enter the cascade, and secondary flow will occur in the blade passages as a result of the pressure difference between the suction and pressure surfaces of the blades. While this departs from strictly two-dimensional flow, it provides better simulation of what occurs in the compressor.

III. GENERAL ARRANGEMENT

Figures 5, 6, and 7 show the cascade installation. A 4 inch gate valve provides tight shut-off, and a Fischer (Type 310-32) automatic control valve is used to operate the tunnel. Design operating pressure in the settling chamber (which is used as control pressure) is 50 lbf/in^2. A tubular flow straightener is used to remove the swirl induced by pipe bends between the control valve and the settling chamber. In the settling chamber a flat, perforated plate diffuser is followed by a screen to reduce turbulence and promote flow uniformity.

A short round-to-rectangular transition section, between the settling chamber and supersonic nozzle, changes the flow area from 10 inches diameter to a rectangular section approximately 2 inches by 4.5 inches. The two-dimensional supersonic nozzle was designed to produce a uniform Mach number of 1.4.

The arrangement of the test section of the cascade is shown in Figure 4. The blades are attached to the test section windows which can be rotated over a small range of incidence (±3 deg). O-ring seals are provided to prevent leakages between sections containing different pressure levels.

Plexiglass windows are provided to allow Schlieren observation and photography of the flow field. Plexiglass

was chosen due to its availability and ease of machining. Stresses in plexiglass are apparent in the Schlieren photographs but need not obscure the main features of interest in the flow field. Schlieren quality glass can be substituted if needed at a later date.

The by-pass exhaust and main exhaust lines duct the flow to the exterior of the building. A butterfly valve is located in the main exhaust line to control the back pressure for the cascade. The plate of the butterfly valve is smaller than the pipe diameter in order to avoid a complete shut-off of the flow.

IV. DESIGN DETAILS

A. NOZZLE THROAT AND TEST SECTION SIZE

The test section flow area is given by $A_1 = H_1 B_1$ where, from the geometry,

$$H_1 = (N + 2f)\ s\ z \cos(\gamma)$$

and

$$B_1 = c\ z\ AR$$

Consequently,

$$(N + 2f) = A^*/[c\ AR\ s\ z^2 \cos(\gamma) / (A_1/A^*)] \qquad (1)$$

At the design Mach number of 1.4, $A_1/A^* = 1.1149$. The values of c, s and γ were given by the geometry of the compressor rotor blading (Table I), and AR = 1 was desired in order to limit blade bending stresses. The throat area of the test nozzle which gave a 2 minute run time was 7.98 in^2. Also, the minimum acceptable scale factor was considered to be about 0.7, in order that the blade sections not be too thin to fabricate properly. Equation (1) then gave (N + 2f) = 7 which would give 5 blade passages (6 blades) for the main flow and 2 full passages exhausting separately. Since a full passage at each end of the cascade was not needed, a one-quarter passage was used, thereby reducing the test section and throat dimensions and increasing the available run time. The nozzle throat area was then 6.27 in^2. At

design stagnation pressure (50 lbf/in^2) and temperature (520^{o}R), the flow rate is 7.3 lb/sec.

Reynolds number similarity was considered. At a rotor relative Mach number of 1.4 the Reynolds number based on chord length for the rotor blades in the compressor is 1.85×10^6. In the cascade at a scale factor of 0.7 the Reynolds number, based on chord length, is 2.58×10^6. Differences in turbulence levels and vibrations between the compressor and the cascade make this difference in Reynolds numbers unimportant for the flow away from the side walls. Of greater importance is the difference in the wall boundary layers at the ends of the blades.

B. SUPPLY PIPING AND SETTLING CHAMBER

It is recommended (Ref. 9) that the velocity in the settling chamber be between 10 and 100 feet per second. A chamber diameter of 10.15 in. was computed to give an average velocity of 50 ft/sec at design conditions, which allowed the use of 10 inch diameter commercial steel pipe.

The existing supply piping from the storage tank to the control valve was 4 inch diameter. Six inch diameter pipe was used from the control valve to the settling chamber to maintain a low pressure drop. A tube-bundle (using 1 inch diameter tubing) was installed as a flow straightener at the end of the supply pipe.

A flat, perforated plate was used at the end of a diverging section to act as a diffuser and decelerate the flow

entering the settling chamber. Reference 9 recommends that the design value for the pressure drop across the perforated plate be approximately equal to the dynamic pressure of the flow in the inlet pipe. Three different plates were available with characteristics as shown in Table II. The data correlations and method given in Ref. 10 were used to calculate the pressure drop across each plate for design conditions in the settling chamber. Values of the plate coefficient for pitch-to-hole-diameter ratios of less than 2 were extrapolated from the given data. The results are shown in Table II. Since the dynamic pressure at the inlet was 0.59 lbf/in^2, all three plates were acceptable on the basis of pressure drop. However, Plate A was chosen since the jets from the smaller holes would require a shorter distance downstream in which to decay. A wire screen was positioned downstream of the diffuser plate to generate small scale turbulence and promote mixing and subsequent turbulence decay in the length available ahead of the nozzle.

C. TRANSITION SECTION

A transition section was required between the 10 inch diameter settling chamber and the rectangular nozzle entrance of approximately 2 in. by 4.5 in. No documented design procedure for such a section was on hand; and since the requirement was simply to achieve a geometrically smooth, accelerating transition between the two sections, a method was devised to compute a suitable shape. The method is

given in Appendix A. Because of limitations in the available machine facilities a small modification, also described in Appendix A, was required to the computed contour. The transition was first machined as a male form which was placed within a 10 inch pipe section. Epoxy resin was cast into the pipe around the form, which was then removed to leave the required inner contour.

D. SUPERSONIC NOZZLE

The nozzle is a conventional two-dimensional design with parallel side walls and contoured end walls. The coordinates used for the supersonic section were provided by the Naval Surface Weapons Center (NSWC), White Oak, Maryland (Ref. 11). A second design was developed in-house using a program written for the Hewlett-Packard 9830A Calculator. Both designs are detailed in Appendix B. The NSWC contours were used since they were obtained from a well-proved design program. The NSWC coordinates which are listed in Appendix B were based on a test section height of 7 blade passages. The test section height was subsequently changed to 5.5 passages heights, and the nozzle coordinates were scaled by a factor of 5.5/7. The longest of three nozzle designs provided by NSWC was chosen. The nozzle coordinates were corrected to account for the boundary layer displacement thickness on both the end walls and the side walls. The boundary layer displacement thickness was calculated using the expression

$$\delta^* = \frac{1}{67} \left(\frac{\nu_{w1}}{\overline{V}_1 x_1}\right)^{.14} M_1^{.75} x \qquad (2)$$

where the bracketed term is the Reynolds number based on design test section conditions and length from the throat (Ref. 12). With the assumption that the displacement thickness at each axial location is the same on both contoured and side walls, the correction to the contoured wall was made as illustrated in Figure 8. The displacement (Δy) which was added to the contoured walls was given by

$$y = \frac{2y\delta^* + (B - 2\delta^*)\,\delta^*}{B} \qquad (3)$$

The scaled contours corrected for the boundary layer displacement thickness are listed in Table III.

The contour upstream of the throat was obtained as part of the transition section design procedure given in Appendix A. The throat of the nozzle (after scaling) had a radius of curvature of 77.1 inches which was used as an input to the transition section design.

E. BLADE STRESSES AND ATTACHMENT

The blade attachment to the plexiglass side wall sections is shown in Figure 9. Round pins are brazed over tabs at the blade ends, using a low temperature aluminum braze. The pins fit into holes bored into the plexiglass and are retained by set screws such that the end of the blade is flush with the inner wall. This method was chosen as it offered relative

ease in manufacture and assembly and reasonable values of blade stress. It also caused little obstruction in viewing the flow through the cascade.

A pressure distribution across the blade chord typical of subsonic flow was assumed initially for blade stress calculations. The pressure distribution was approximated by a linear variation from the leading to the trailing edge with zero pressure difference at the trailing edge. With this approximation for the loading, a larger pin was required at the front than at the back of the blade. However, the rear pin was made equal in size to the front pin to provide adequate strength for any physically reasonable load distribution.

A summary of the calculations follows:

The lift per unit span was calculated using

$$L/B = C_L \, q_\infty \, c \tag{4}$$

where L is the total lift force on one blade, C_L is the lift coefficient given by

$$C_L = \frac{2}{\sigma} (\tan \beta_1 - \tan \beta_2) \cos \beta_\infty \tag{5}$$

and q_∞ is the relative dynamic pressure given by

$$q_\infty = \frac{1}{2} \rho \, W_\infty^2 \tag{6}$$

The quantities W_∞ and β_∞ are as shown in Figure 10 where

$$\tan \beta_\infty = \frac{1}{2} (\tan \beta_1 + \tan \beta_2) \tag{7}$$

Assuming a perfect gas, in terms of the known upstream conditions, Eq. (6) can be written as

$$q_\infty = \frac{k}{2} P_{t1} M_1^2 \left(1 + \frac{k-1}{2} M_1^2 \right)^{\frac{-k}{k-1}} \left(\frac{\cos^2 \beta_1}{\cos^2 \beta_\infty} \right) \quad (8)$$

If β_2 is known, the lift force can be calculated using Eq. (4), first obtaining β_∞ from Eq. (7), C_L from Eq. (5) and q_∞ from Eq. (8).

β_2 was determined for a range of β_1 using the data correlations given in Chap. 6 of Ref. 13. β_1 for the minimum loss incidence was 63 degrees for the given blade shape, and operation at ±3 degrees from this condition was required. First, using Ref. 13, the deviation angle, δ, was calculated. It can be seen in Fig. 1 that the incidence angle, i, is related by

$$i = \beta_1 - \gamma - \frac{\phi}{2} \quad (9)$$

and since, also from the geometry,

$$\Delta\beta = \beta_1 - \beta_2 = \phi + i - \delta \quad (10)$$

the outlet flow angle is given by

$$\beta_2 = \gamma - \frac{\phi}{2} + \delta \quad (11)$$

The results of calculating β_2 using Eq. (11) and C_L using Eqs. (7) and (5) are listed in Table IV.

The highest C_L (at β_2 = 66 deg) was used to calculate the maximum load on the blading. From Eq. (4) the total

blade lift was then L = 22.91 lbf. If the lift is distributed linearly in the direction of the chord and is supported by forces L_1 and L_2 at the front and rear pins respectively, then the situation is that shown in Figure 11, where P is the pressure loading at the leading edge. The total lift force is given by L = 1/2 P c B or P = 2 L/cB. For the present design with AR = 1,

$$P = \frac{2L}{c^2} \tag{12}$$

The pins are separated by a fraction, x, of the chord and are located symmetrically with respect to the mid-chord point. Equilibrium of the forces on all four pins requires that

$$L_2 = \frac{L}{2} - L_1 \tag{13}$$

and summing moments about point B gives

$$L_1 (1 + x) + L_2 (1 - x) = \frac{2L}{3} \tag{14}$$

The loads at the pins are therefore given by

$$\frac{L_1}{L} = \frac{1}{4} + \frac{1}{12x}$$

and (15)

$$\frac{L_2}{L} = \frac{1}{4} - \frac{1}{12x}$$

The section of blade between the front pins was treated as a beam fixed at each end and carrying a uniformly distributed load. Using Eq. (15) and assuming that each pair of pins supports half the blade span, for the front pins the

load per unit span, w_1, is given by

$$w_1 = \frac{2L_1}{B} = \frac{L}{B}\left(\frac{1}{2} + \frac{1}{6x}\right) \qquad (16)$$

Several values of x were chosen, and the bending stresses were calculated in pins of different diameter (and therefore tab width). The variation in the area moment and centroid of the tab section with x and the variation in the pin radius were included in the calculation. A value of x = .5 and a pin radius of .15 inch were selected. For these dimensions, the tensile stress on the lower portion of the tab was calculated to be 20,000 lbf/in^2. The material selected for the blading was 7075-T6 Aluminum, which has a yield stress of approximately 65,000 lbf/in^2.

F. BY-PASS DUCTING

Part of the flow is removed from the cascade end walls by a scoop arrangement as shown in Figure 12. The pressure drop was calculated for this section using a simplified model of the process. The assumptions and simplifications are as follows:

1. Uniform flow at a Mach number of 1.4, stagnation pressure of 50 lbf/in^2, and stagnation temperature of 520°R decelerates due to a normal shock at 1.

2. The decelerated flow turns without losses, sustaining losses in the sudden expansion process. The sudden expansion analysis is given in Appendix C.

3. The flow is steady with perfect gas behavior.

4. Gravitational forces are neglected.

For a normal shock from upstream conditions (denoted by x) at $M_x = 1.4$, the downstream conditions (denoted by y), which will be the conditions at station 1 in the analysis from Appendix C, are:

$$M_y = .74$$

$$P_y/P_x = 2.12$$

$$P_{oy}/P_x = 3.05$$

Given that

$$P_t = 50 \text{ lbf/in}^2$$

and at $M_x = 1.4$

$$P_x/P_{tx} = .314$$

then

$$M_y = .74$$

$$P_{oy} = P_t(P_x/P_t)(P_{oy}/P_x) = 47.88 \text{ lbf/in}^2$$

and

$$P_y = P_t(P_x/P_t)(P_y/P_x) = 33.28 \text{ lbf/in}^2.$$

Using $P_1 = 33.28 \text{ lbf/in}^2$, $V_1 = 785$ ft/sec (for $M_y = .74$), and $A_1 = .47 \text{ in}^2$ and $A_2 = 3.39 \text{ in}^2$ (from the cascade geometry) in Eqs. C-9 and C-7 of Appendix C, the static pressure at station 2 was $P_2 = 33.8 \text{ lbf/in}^2$.

The expansion losses calculated here are likely to be greater than the losses which were neglected in the flow

turning. The by-pass duct is vented downstream at atmospheric pressure, so that for these conditions there is sufficient pressure remaining to operate the by-pass exhaust properly. (It is noted that during the starting process, the flow conditions at the scoop entrance will be considerably different and the pressure level difficult to predict.)

V. CONCLUDING COMMENTS

This document reports the initial design and fabrication of a new, small test facility. In order to satisfy the goal of simulating an existing rotor tip design at low cost, a simple rather than elaborate and more flexible apparatus was built.

Initial tests are required to verify:

(1) nozzle flow uniformity,

(2) proper operation of by-pass ducts,

(3) adequate strength of blade attachments, and

(4) proper flow conditions in the cascade.

The slotted section will be added to the end of the upper nozzle block after completion of items (1) and (2). Slots will be machined into the nozzle block and vented through to the upper by-pass duct. This arrangement should also assist in starting the flow when the blading is installed.

Instrumentation for the initial tests will consist of an impact pressure rake and static pressure taps in the nozzle and test section walls. A window section containing pressure taps simulating the pattern of pressure taps in the compressor will be added when Schlieren observations of the flow in the cascade have been made and are satisfactory.

TABLE I. Compressor Rotor Tip and Cascade Blade Data

	Rotor Tip	Cascade
Scale Factor (z)	1	0.7
Stagger Angle (γ)	59 deg, 44 min, 35 sec	59 deg, 44 min, 35 sec
Camber Angle (ϕ)	4.7 deg	4.7 deg
Blade Spacing (s), in	1.920	1.344
Blade Chord (c), in	2.688	1.882
Leading edge and trailing edge radii, in	.005	.003
Suction side radius, in	16.33	11.431
Maximum thickness, in	.065	.045

TABLE II. Perforated Plate Data

	Plate A	Plate B	Plate C
Thickness, in.	.1875	.1875	.1875
Hole Diameter, in.	.25	.625	.75
Pitch/Hole Diameter	1.5	1.4	1.33
Thickness/Hole Diameter	.75	.3	.25
ΔP, lbf/in^2	.49	.57	.44

Holes are on an equilateral triangle pitch. Pitch is the distance between hole centers. ΔP was calculated for the settling chamber flow conditions.

TABLE III. Supersonic Nozzle Coordinates

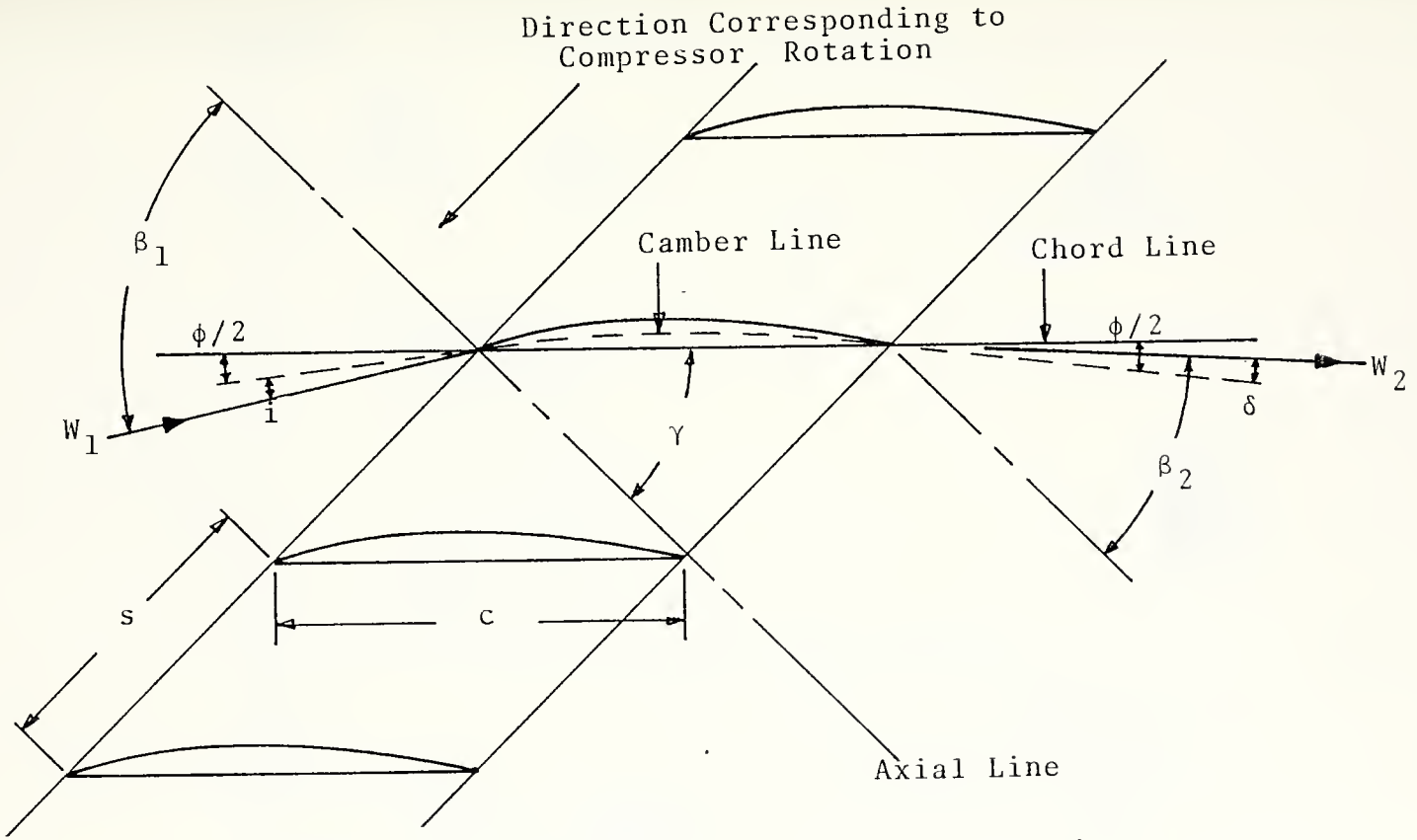
I	X(INCHES)	Y(INCHES)	I	X(INCHES)	Y(INCHES)
1	0.000	2.086	21	10.207	3.075
2	0.500	2.321	22	10.801	3.057
3	1.000	2.537	23	11.321	3.044
4	1.500	2.728	24	11.948	3.032
5	2.000	2.891	25	12.584	3.023
6	2.500	3.023	26	13.230	3.016
7	3.000	3.123	27	13.883	3.012
8	3.500	3.192	28	14.429	3.008
9	4.000	3.233	29	14.913	3.005
10	4.500	3.250	30	15.379	3.002
11	5.000	3.254	31	15.881	2.999
12	5.506	3.247	32	16.365	2.996
13	5.933	3.238	33	16.849	2.993
14	6.440	3.223	34	17.333	2.991
15	7.017	3.202	35	17.817	2.988
16	7.436	3.186	36	18.301	2.985
17	7.881	3.167	37	18.786	2.982
18	8.597	3.137	38	19.270	2.979
19	9.106	3.116	39	19.754	2.977
20	9.642	3.095	40	20.238	2.974

COORDINATES 1 through 28 ARE FOR THE BOTTOM NOZZLE BLOCK.
COORDINATES 1 through 40 ARE FOR THE TOP NOZZLE BLOCK.

- -

TABLE IV. C_L Tabulated for Various Values of β_2

β_2 (deg)	C_L
60	.057
61	.106
62	.156
63	.208
64	.262
65	.318
66	.376



β_1 = relative inlet flow angle
W_1 = inlet relative velocity
β_2 = relative exit flow angle
W_2 = exit relative velocity
γ = stagger angle
s = blade spacing
c = blade chord
ϕ = camber angle
i = incidence angle
δ = deviation angle

Figure 1. Cascade Notation

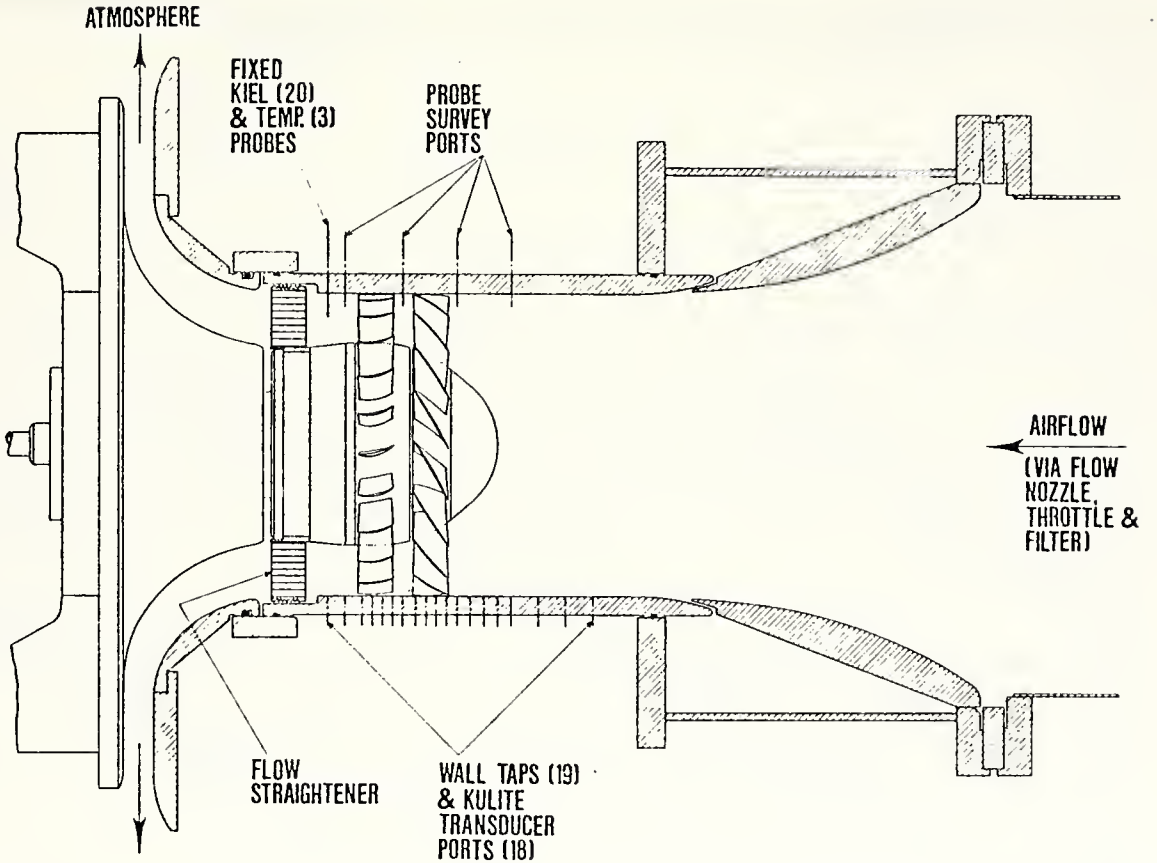


Figure 2. Transonic Compressor

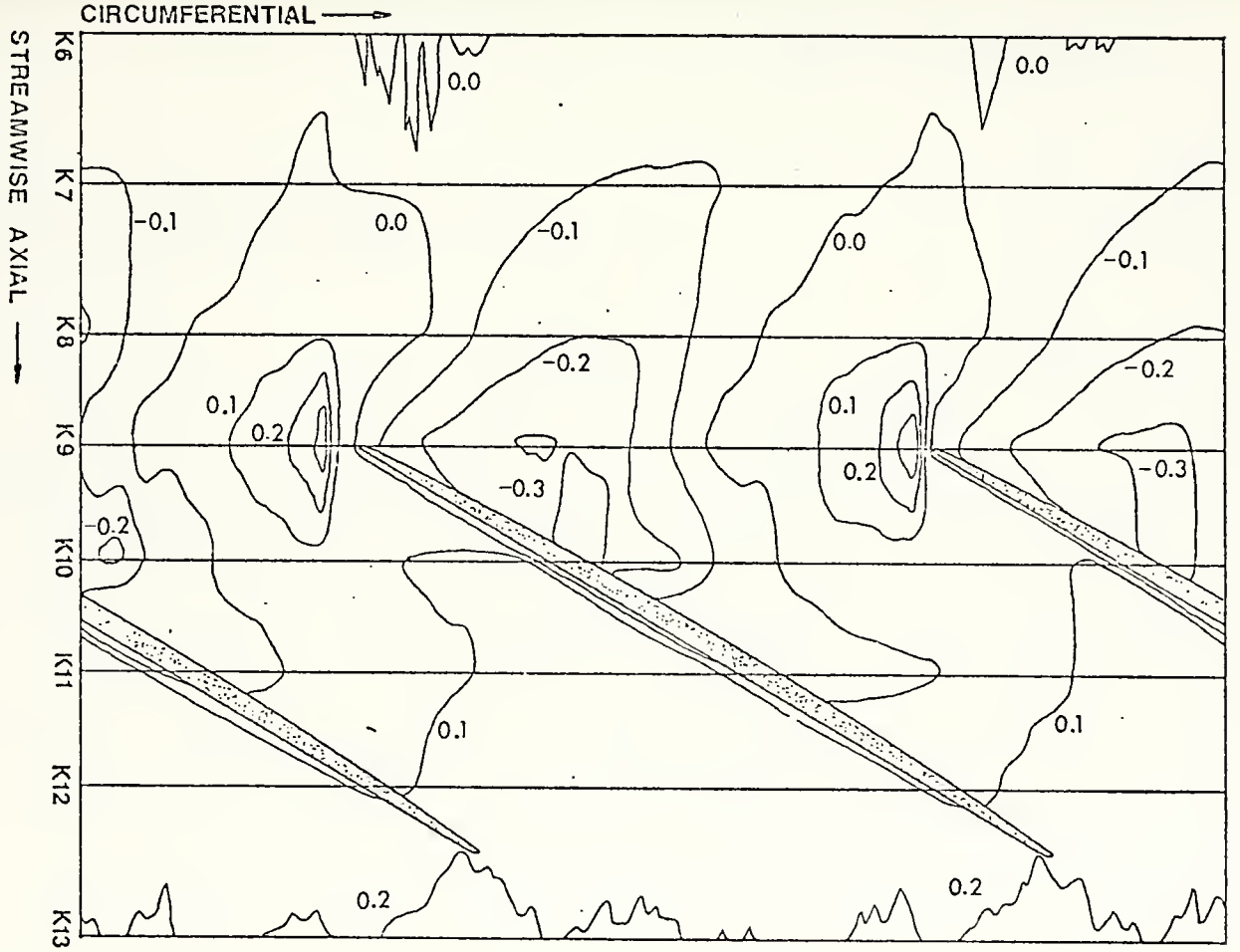


Figure 3. Transonic Compressor Case Wall Pressure Map

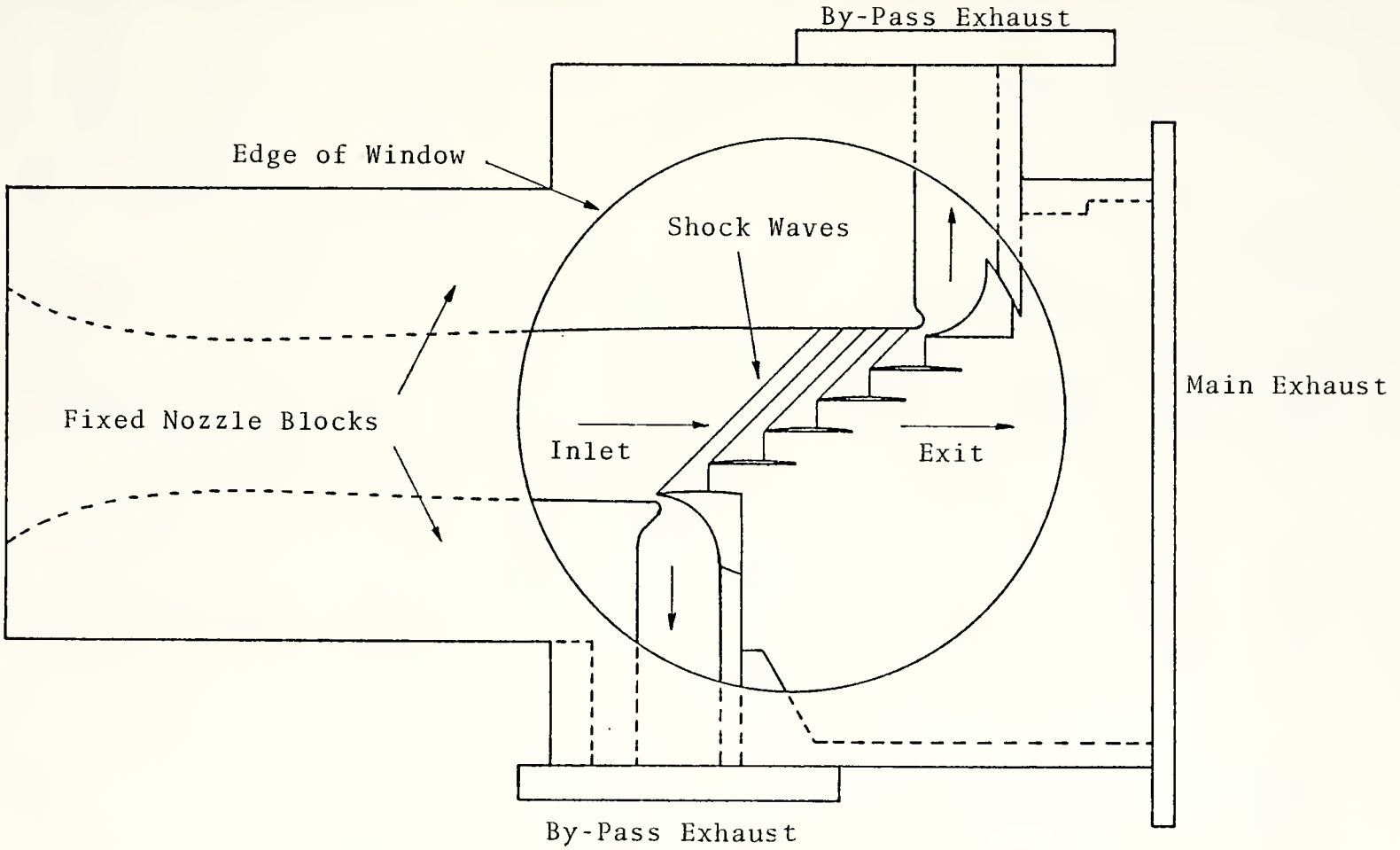


Figure 4. Cascade Arrangement

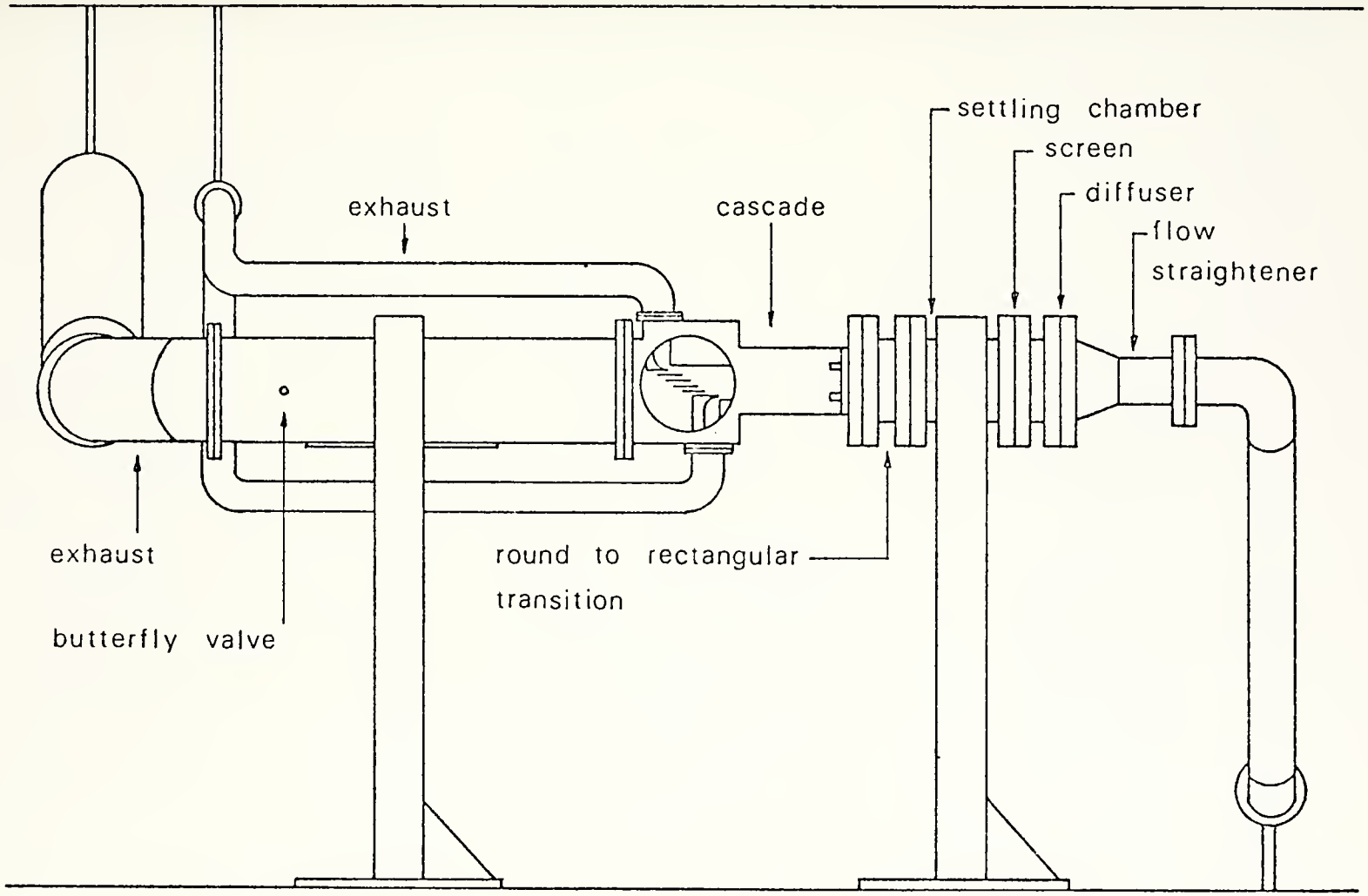


Figure 5. Cascade Wind Tunnel

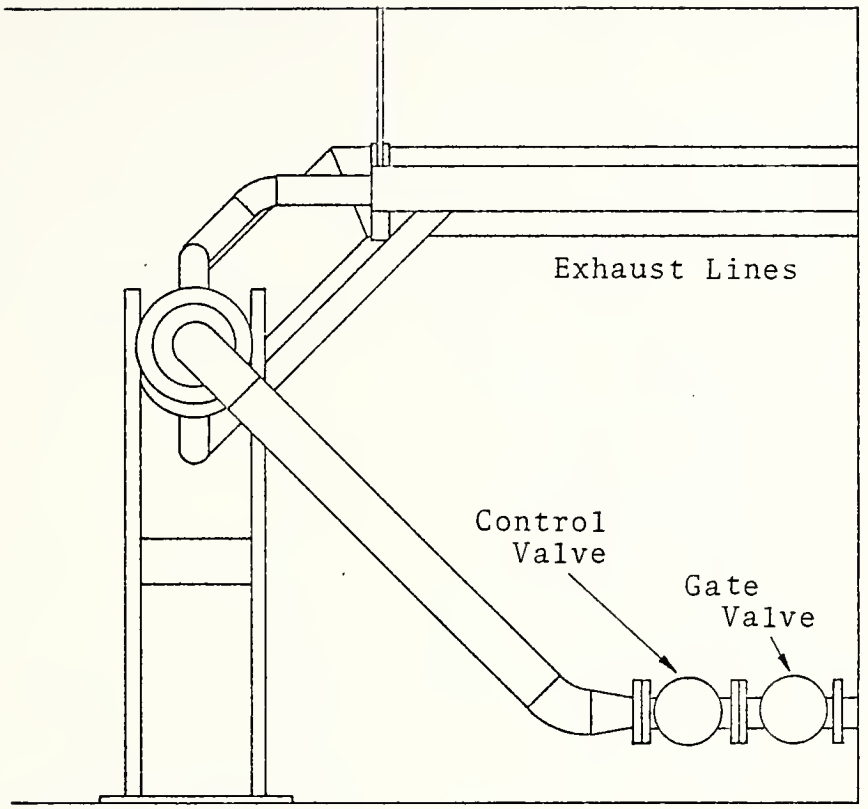


Figure 6. Cascade Wind Tunnel (from upstream)

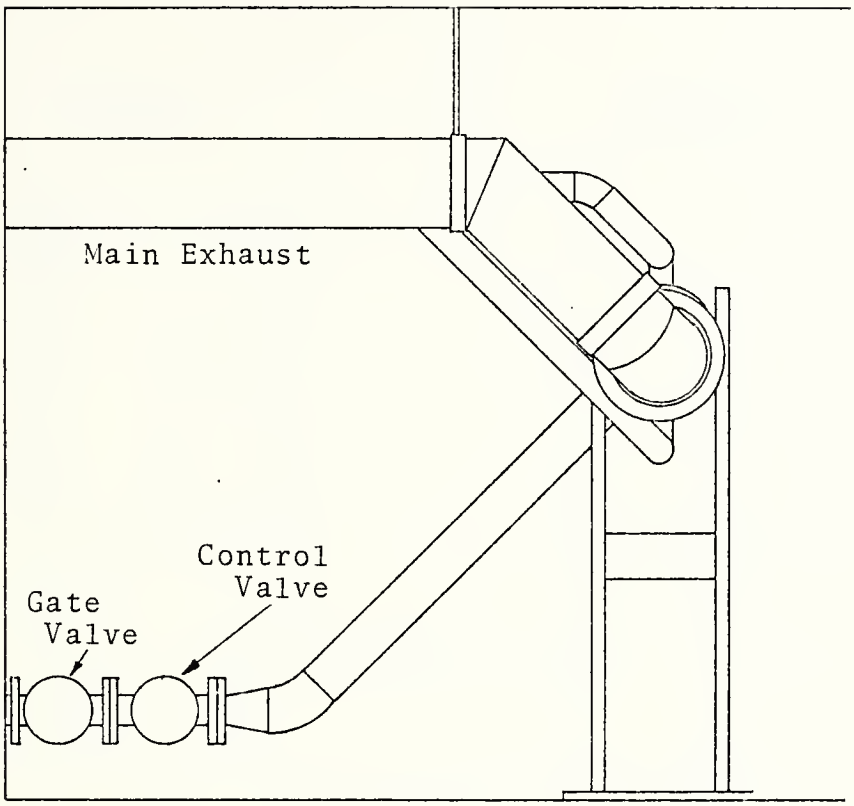
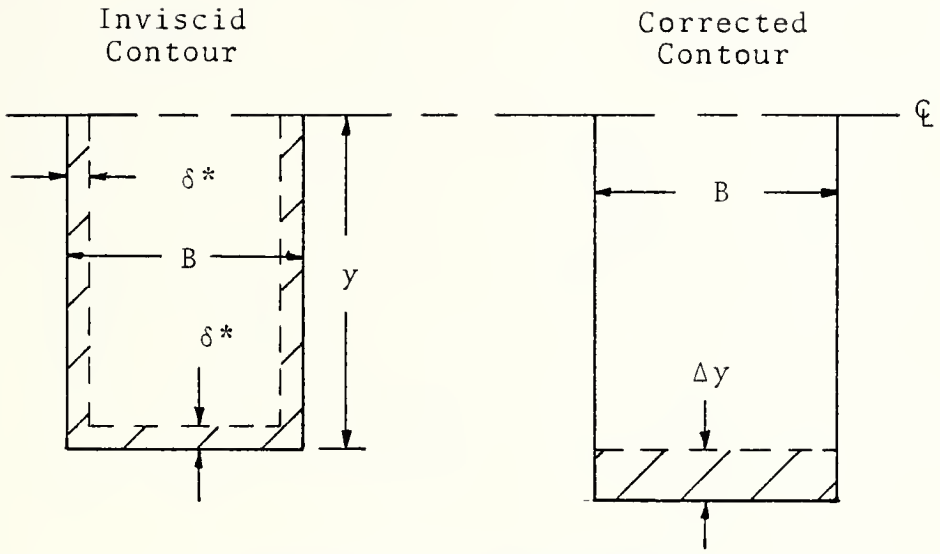


Figure 7. Cascade Wind Tunnel (from downstream)



Shaded areas are equal.

Figure 8. Nozzle Wall Correction for Boundary Layer

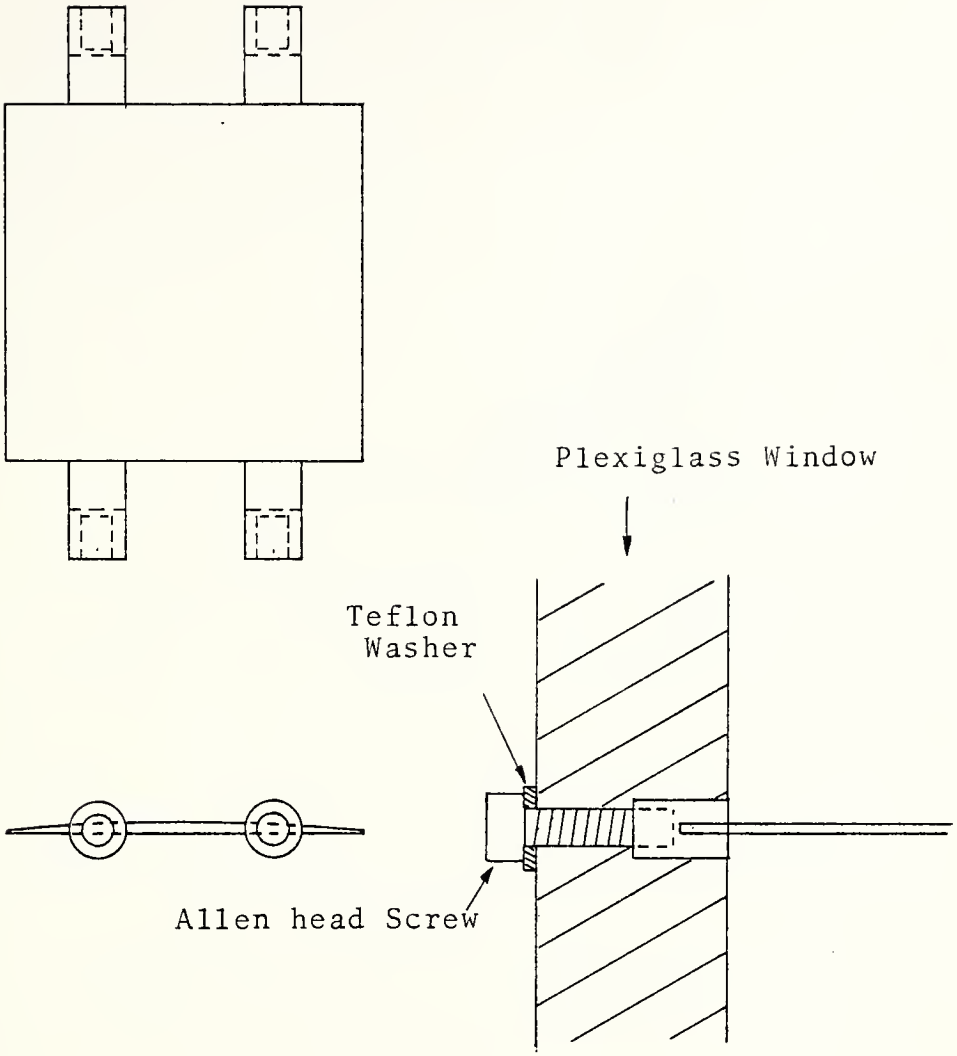


Figure 9. Method of Blade Attachment

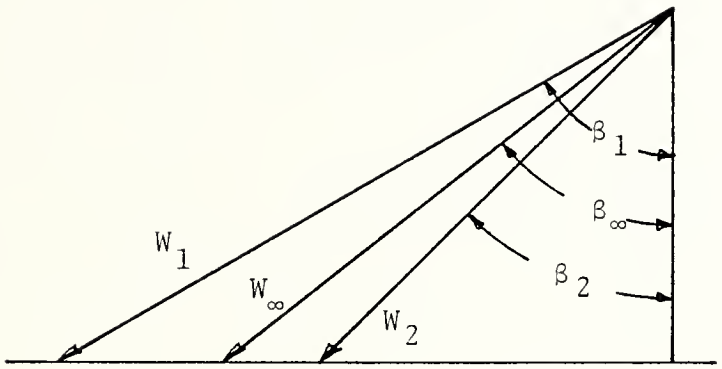


Figure 10. Definition of β_∞ and W_∞

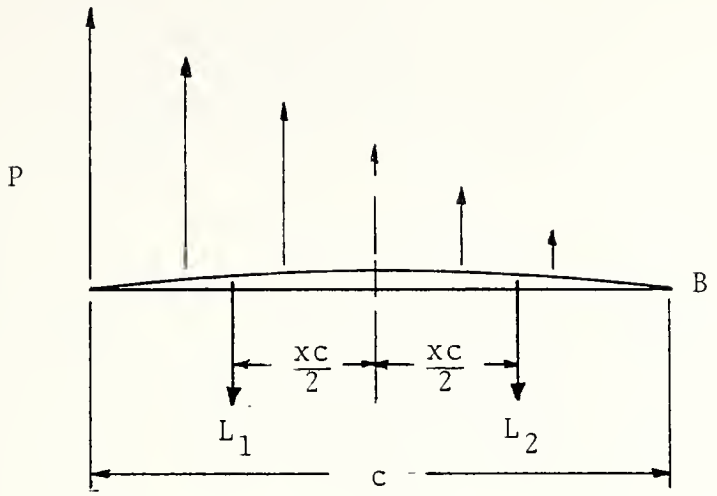


Figure 11. Assumed Blade Loading

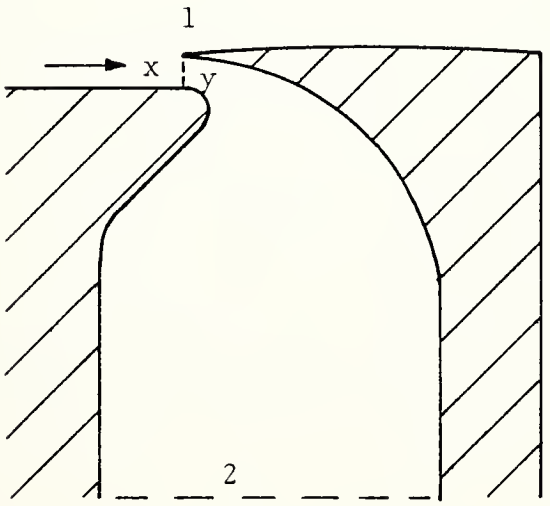


Figure 12. By-Pass Scoop

APPENDIX A

TRANSITION SECTION DESIGN

1. Approach

A transition from a round section to a smaller rectangular section was designed to join the settling chamber to the nozzle blocks. At the upstream end of the section the wall is cylindrical. At the downstream joint, where the cross section is rectangular, the side walls are parallel, and the upper and lower contours are converging towards the nozzle throat. Polynomial expressions were assumed for the wall contours in the horizontal and vertical planes, and the required boundary conditions were satisfied to determine the unknown coefficients in the polynomials. Then, a radius was added as shown in Figure A-3 so that the typical cross section at any station was a rectangle with rounded corners. The condition applied to calculate the corner radius was that the derivative should be zero at the upstream end, and that the rate of change of area at the exit should match that of the nozzle blocks.

2. Method

For the vertical plane profile shown in Figure A-1, the contour is given by

$$w = a_1 z^2 + b_1 z^3 + c_1 z^4 + d_1 z^5$$

and satisfies the conditions that

$$w(0) = 0$$

$$w(L) = R_o - H/2$$

$$\frac{dw}{dz}(0) = 0$$

$$\frac{dw}{dz}(L) = 0$$

$$\frac{d^2w}{dz^2}(z_o) = 0 \text{ (inflection point is at the joint)}$$

$$\frac{d^2w}{dz^2}(L) = -1/r^*$$

From these conditions:

$$d_1 = \frac{[36(R_o-H/2)-6L^2/r^*][(L-z_o)-(L^2-z_o^2)/L]+[12(R_o-H/2)(L-z_o)-L^3/r^*]}{36L^5[(L-z_o)-(L^2-z_o^2)/L]-L^5[18(L-z_o)-20(L^3-z_o^3)/L^2]}$$

$$c_1 = [6(R_o-H/2)-L^2/r^*]/2L^4 - 3d_1L$$

$$b_1 = [-2(R_o-H/2)/L^3] - 2c_1L - 3d_1L^3$$

$$a_1 = -1/2(3b_1L + 4c_1L^2 + 5d_1L^3)$$

For the horizontal plane profile shown in Figure A-1, the contour is given by:

$$v = a_2z^2 + b_2z^3 + c_2z^4$$

and satisfies the conditions that,

$$v(0) = 0$$

$$v(z_o) = R_o - B/2$$

$$\frac{dv}{dz}(0) = 0$$

$$\frac{dv}{dz}(z_o) = 0$$

$$\frac{d^2v}{dz^2}(z_o) = 0$$

From these conditions:

$$a_2 = 6/z_o^2(R_o - B/2)$$

$$b_2 = -4/3(a_2/z_o)$$

$$c_2 = a_2/(2z_o)$$

The corner radius is given by

$$(r/R_o)^2 = a_3 + b_3(z/z_o) + c_3 (z/z_o)^2 + d_3 (z/z_o)^3$$

and satisfies the conditions that:

$$\frac{r(0)}{R_o} = 1$$

$$\frac{r(z_o)}{R_o} = 0$$

$$\frac{dr}{dz}(0) = 0$$

and

$$\frac{dA}{dz} = \frac{d}{dz}[2B(R_o - w)] \quad \text{at } z = z_o$$

where

$$A = 4(R_o - v)(R_o - w) - r^2(4 - \pi)$$

From these conditions,

$$a_3 = 1$$

$$b_3 = 0$$

$$c_3 = -3/(z_o/R_o)^3$$

$$d_3 = 2/(z_o/R_o)^3$$

3. Design Program

The equations in the preceding paragraphs were used in a BASIC language program "BLMTH" written for the HP-9830A calculator. The program accepts as inputs the overall length of the transition section to the nozzle throat, the distance required to the joint, the nozzle throat radius, height and width, and the inlet pipe radius. The program calculates the contours in the vertical and horizontal planes, the value of the corner radius as a function of distance, tabulates these values and plots the contours as shown in Figures A-2 and A-3. The data input to obtain these figures were as follows:

pipe radius	= 5 inches
nozzle width	= 1.882 inches
throat height	= 3.331875 inches
radius at the throat	= 77.1 inches
total length of section	= 12 inches
distance to joint	= 6 inches

A listing of the program is given in Table A-I and a tabulated output is given in Table A-II.

4. Modification to the Design

The design developed in the preceding section could not be machined with available equipment and the male form for the section was made in the following way. An axisymmetric form was machined first following the contour computed for the vertical plane. The form was then machined vertically along the sides to produce the contour computed for the horizontal plane. The resulting corners were radiused by hand.

```
10 REM*******BLMTH*******W.J.DEMO******22/2/78
20 REM      PROGRAM TO DESIGN CONTRACTION(BELLMOUTH) FROM
30 REM      ROUND SECTION TO 2-DIM NOZZLE ENTRY.
40 DIM X[101],Y[101],R[101],V[101],W[101],Z[101]
50 MAT X=ZER
60 MAT Y=ZER
70 MAT W=ZER
80 MAT R=ZER
90 MAT Z=ZER
100 MAT V=ZER
110 REM
120 DISP "ENTER PIPE RADIUS IN INCHES";
130 INPUT R0
140 DISP "ENTER NOZZLE WIDTH IN INCHES";
150 INPUT B9
160 DISP "ENTER THROAT HEIGHT IN INCHES";
170 INPUT H9
180 DISP "ENTER RADIUS AT THE THROAT (INS)";
190 INPUT R9
200 REM
210 REM-----ENTER LENGTH AND DIST. TO JOINT
220 REM
230 DISP "ENTER TOTAL LENGTH OF SECTION";
240 INPUT L9
250 DISP "ENTER DIST. TO JOINT IN INCHES";
260 INPUT Z0
270 B8=B9/R0
280 H8=H9/R0
290 R8=R9/R0
300 L8=L9/R0
310 Z8=Z0/R0
320 REM
330 REM-----CALC. CENTERLINE PROFILE-----
340 REM
350 D5=L8-Z8-(L8↑2-Z8↑2)/L8
```

TABLE A-I. "BLMTH" Program Listing

```
360 D0=(36*(1-H8/2)-6*L8*L8/R8)*D5+12*(1-H8/2)*(L8-Z8)-(L8↑3)/R8
370 D0=D0/((36*D5-18*(L8-Z8)+20*(L8↑3-Z8↑3)/L8↑2)*L8↑5)
380 C0=(6*(1-H8/2)-(L8↑2)/R8)/(2*L8↑4)-3*D0*L8
390 B0=-2*(1-H8/2)/L8↑3-2*C0*L8-3*D0*L8*L8
400 A0=-(3*B0*L8*L8+4*C0*L8↑3+5*D0*L8↑4)/(2*L8)
410 DEF FNZ(Z5)=A0*Z5↑2+B0*Z5↑3+C0*Z5↑4+D0*Z5↑5
420 REM
430 REM-----CALC. JOINT-----
440 P8=-(3*B0+4*C0*L8+5*D0*L8↑2)*L8
450 W8=FNZ(Z8)
460 V8=FNV(1)
470 D8=SQR((B8/2)↑2+(1-W8)↑2)
480 REM-----CALCULATE HORIZONTAL CENTERLINE PROFILE
490 REM
500 DEF FNV(Z5)=6*(1-B8/2)*Z5*Z5*(1-4*Z5/3+Z5*Z5/2)
510 REM-----COMPUTE CROSS SECTION PROPERTIES
520 REM
530 WRITE (15,540)
540 FORMAT /,/,/,/,/,/,20X,"BELLMOUTH CONTOUR DIMENSIONS"
550 WRITE (15,560)" HALF HT. "," HALF WTH."
560 FORMAT 10X,"  AX.DIST.","   CENTER COORDS.   ","  RADIUS  ",F10.0
570 PRINT
580 REM
590 I=1
600 DISP "ENTER NO.OF STEPS FOR COORDS.";
610 INPUT N9
620 FOR N8=0 TO N9 STEP 1
630 Z1=Z8*N8/N9
640 Z9=Z1/Z8
650 W1=FNZ(Z1)
660 V1=FNV(Z9)
670 V7=V1/V8
680 W7=W1/W8
690 F0=1.5
700 R1=(1-(V7+F0*W7)/(1+F0))↑2
```

TABLE A-I. "BLMTH" Program Listing (Continued)

```
710 X1=1-V1-R1
720 Y1=1-W1-R1
730 Z[I]=Z1*R0
740 X[I]=X1*R0
750 Y[I]=Y1*R0
760 R[I]=R1*R0
770 V[I]=(1-V1)*R0
780 W[I]=(1-W1)*R0
790 WRITE (15,800)I,Z[I],X[I],Y[I],R[I],W[I],V[I]
800 FORMAT 5X,F3.0,2X,6F10.6
810 I=I+1
820 NEXT N8
830 I=N9+1
840 FOR N8=(N9+1) TO (2*N9)
850 Z1=Z8*N8/N9
860 Z[I]=Z1*R0
870 W[I]=(1-FNZ(Z1))*R0
880 WRITE (15,890)I,Z[I],W[I]
890 FORMAT 5X,F3.0,2X,F10.6,30X,F10.6
900 I=I+1
910 NEXT N8
920 DISP "<CONT><F0> FOR PLOT";
930 STOP
```

TABLE A-I. "BLMTH" Program Listing (Continued)

```
10 REM-----*****BLMTH1**KEY<F0>----W.J.DEMO   -----23/2/78
20 REM-----KEY PROGRAM TO PLOT BELLMOUTH CONTOURS.
30 REM
40 DISP "SET PLOTTER 10 BY 7(HIGH)-<CONT>";
50 STOP
60 SCALE -0.4,3.6,-0.4,2.4
70 LABEL (*,1.7,2,0,0.7)
80 DISP "AXES?----1=YES, 0=NO";
90 INPUT N0
100 IF N0=0 THEN 230
110 XAXIS 1,0.2,0,3.5
120 YAXIS 0,0.2,0,2
130 FOR Y=0 TO 2
140 PLOT 0,Y,1
150 CPLOT -3,-0.3
160 LABEL (*)Y-1
170 NEXT Y
180 FOR X=1 TO 3
190 PLOT X,1,1
200 CPLOT -1,-1
210 LABEL (*)X
220 NEXT X
230 FOR Z1=0 TO L8 STEP L8/100
240 PLOT Z1,FNZ(Z1)
250 NEXT Z1
260 PEN
270 CPLOT 1,0
280 LABEL (*)"WALL PROFILE IN"
290 LABEL (*)"THE RADIAL PLANE"
300 LABEL (*)"PARALLEL TO THE"
310 LABEL (*)"NOZZLE SIDE WALLS"
320 FOR Z1=P1 TO Z8 STEP ((Z8-P1)/10)
330 PLOT Z1,FNB(Z1)
340 NEXT Z1
350 STOP
```

TABLE A-I. "BLMTH" Program Listing (Continued)

```
360 FOR Z1=0 TO Z8 STEP Z8/100
370 Z9=Z1/Z8
380 PLOT Z1,2-FNV(Z9)
390 NEXT Z1
400 PEN
410 CPLOT 1,3
420 LABEL (*)"WALL PROFILE IN"
430 LABEL (*)"THE RADIAL PLANE"
440 LABEL (*)"NORMAL TO THE"
450 LABEL (*)"NOZZLE SIDE WALLS"
460 DISP "<CONT>< F1 > FOR SECTION PLOTS";
470 STOP


10 REM-----*****BLMTH1***** KEY < F1 >---
20 REM-----TO PLOT BELLMOUTH CROSS SECTIONS
30 DISP "SET PLOTTER 10.5 BY 10.5,<CONT>";
40 STOP
50 SCALE -5.01,5.01,-5.01,5.01
60 LABEL (*,1.7,2,0,1)
70 DISP "AXES?,  1=YES  0=NO";
80 INPUT N0
90 IF N0=0 THEN 120
100 XAXIS 0,1,-5,5
110 YAXIS 0,1,-5,5
120 REM------BEGIN PLOT
130 DISP "INPUT NO. OF SECTION";
140 INPUT I
150 X1=X[I]
160 Y1=Y[I]
170 T0=0
180 GOSUB 310
190 PLOT -X9,Y9
200 X1=-X1
210 GOSUB 310
```

TABLE A-I. "BLMTH" Program Listing (Continued)

```
220 PLOT X9,-Y9
230 Y1=-Y1
240 GOSUB 310
250 PLOT -X9,Y9
260 X1=-X1
270 GOSUB 310
280 PLOT X9,-Y9
290 PEN
300 GOTO 130
310 FOR T1=T0 TO T0+PI/2 STEP PI/100
320 X9=X1+R[I]*COS(T1)
330 Y9=Y1+R[I]*SINT1
340 PLOT X9,Y9
350 NEXT T1
360 T0=T1
370 RETURN
```

TABLE A-I. "BLMTH" Program Listing (Continued)

BELLMOUTH CONTOUR DIMENSIONS

	AX.DIST.	CENTER COORDS.		RADIUS	HALF HT.	HALF WTH.
1	0.000000	0.000000	0.000000	5.000000	5.000000	5.000000
2	1.000000	0.051301	0.565359	4.413137	4.978496	4.464438
3	2.000000	0.224869	1.748188	3.121464	4.869652	3.346383
4	3.000000	0.440068	2.869088	1.769369	4.638457	2.209438
5	4.000000	0.642923	3.535810	0.749077	4.284886	1.392000
6	5.000000	0.831498	3.659881	0.175273	3.835155	1.006771
7	6.000000	0.941000	3.332969	0.000000	3.332969	0.941000
7	7.000000				2.830783	
8	8.000000				2.381051	
9	9.000000				2.027480	
10	10.000000				1.796286	
11	11.000000				1.687442	
12	12.000000				1.665938	

TABLE A-II. Transition Section Contour

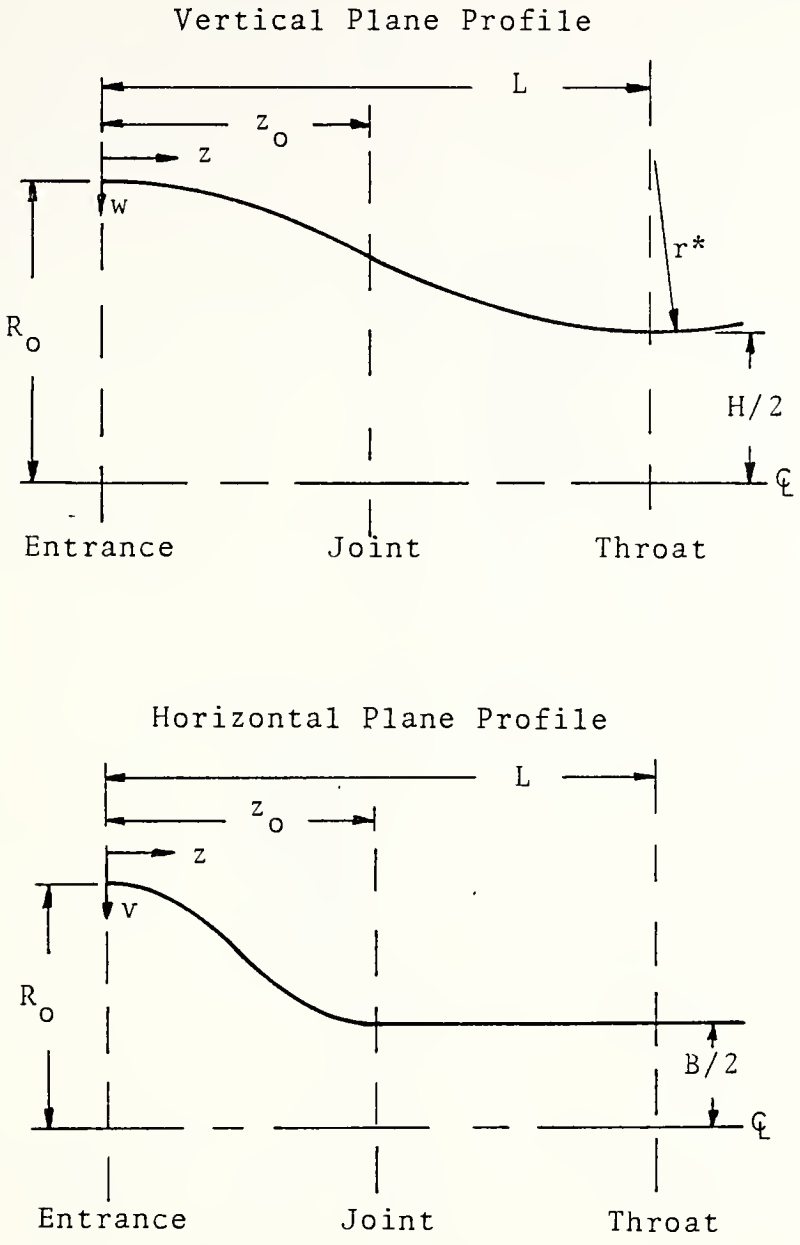


Figure A-1. Vertical and Horizontal Plane Profiles for the Transition Section.

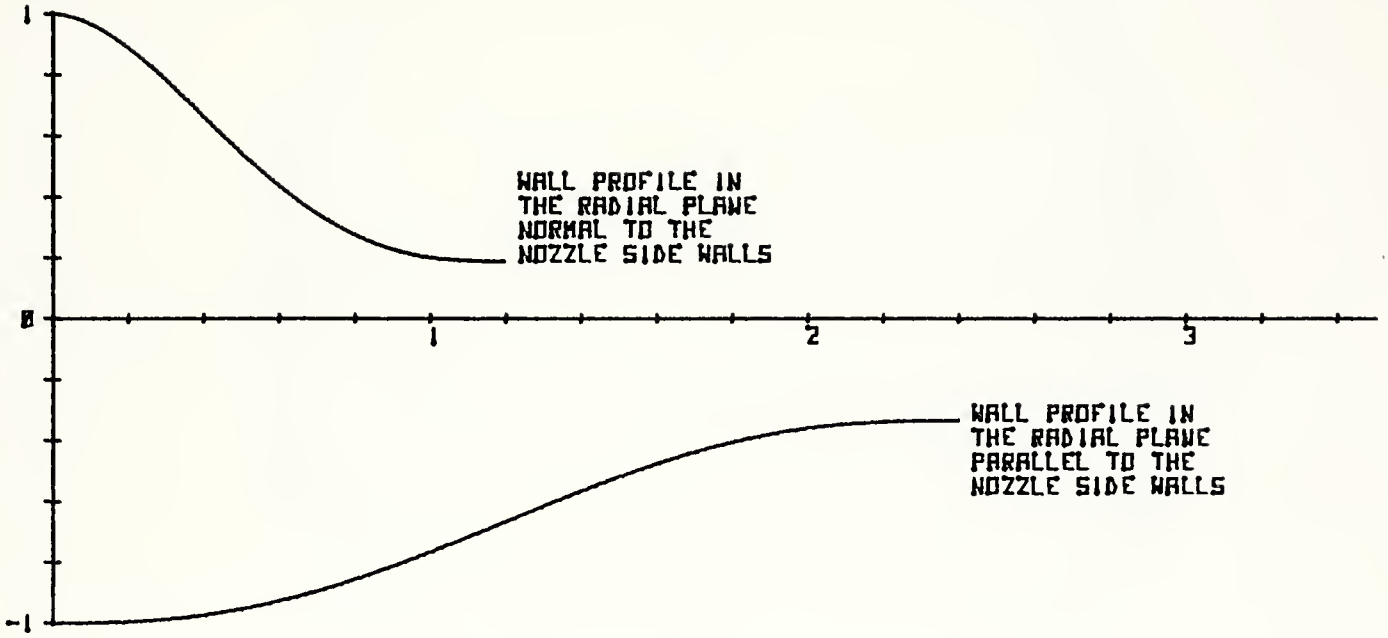


Figure A-2. Horizontal and Vertical Profiles from "BLMTH"

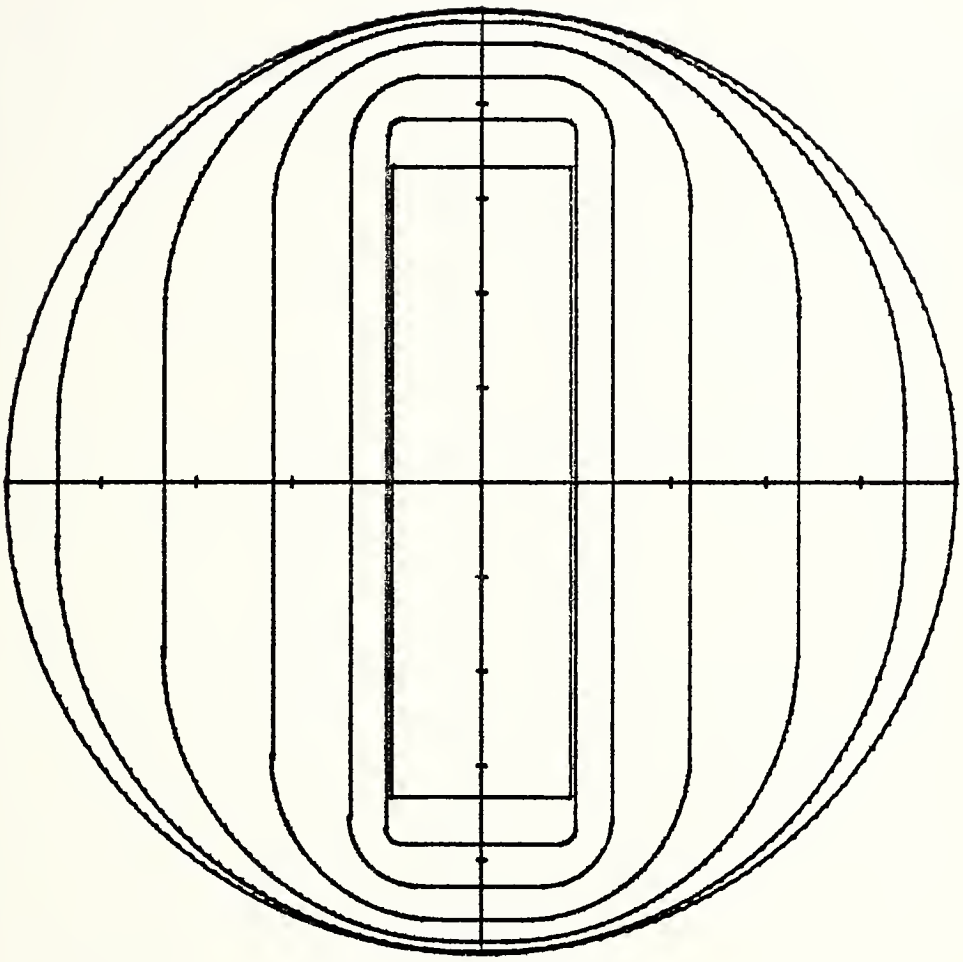


Figure A-3. Cross Section Contour from "BLMTH"

APPENDIX B

SUPERSONIC NOZZLE DESIGN

1. NSWC Program

Four designs were calculated. Inputs common to each design were, Mach number = 1.4, Exit half-height = 2.364 in, and Gas = air (k = 1.4). Contours were calculated for a (calculated) minimum length of 4.212 inches and lengths of 6, 9, and 12 inches. The radii at the throats were -3.3×10^4, 20.9, 37.7, and 60.6 inches respectively. The 12 inch nozzle was used in the cascade design. The computer printout is given in TABLE B-I.

2. Method of Characteristics Program

The differential equations for supersonic flow in two dimensions can be solved by the method of characteristics, and the contours of supersonic nozzles can be obtained by several techniques which use this approach. The technique outlined in Ref. 9 was followed here.

a. Method

The notation used is shown in Figure B-1. Each region is labeled (a,b), where 'a' denotes the number of degrees of turning produced by waves from the upper surface and 'b' denotes the number of degrees of turning produced by waves from the lower surface. The local flow angle is then (a-b) and the total turning (a+b). The angle with

respect to the horizontal of a right running characteristic is the difference between the Mach angle and the local flow angle. The angle with respect to the horizontal of a left running characteristic is the sum of the Mach angle and the local flow angle. For each region the Mach number must be found. The procedure is as follows:

(1) The total turning (U) produced by the flow is determined from the design Mach number (M_1) using

$$U = \sqrt{\frac{k+1}{k-1}} \tan^{-1} \sqrt{\frac{(k-1)}{(k+1)}(M_1^2-1)} \tan^{-1} \sqrt{M_1^2-1} \quad \text{(B-1)}$$

(2) The degree of turning that each step in the straight line construction of the contour will produce is calculated from

$$D = U/2N \quad \text{(B-2)}$$

where N is the number of characteristics used. The length of each of these initial steps is fixed as X1. Newton's method is then used to solve Eq. B-1 for the Mach number in each region.

(3) The Mach angle (μ) in each region is found using

$$\mu = \tan^{-1} \frac{1}{\sqrt{M^2-1}} \quad \text{(B-3)}$$

(4) The angle of the characteristic itself is taken as the average of the characteristic angles for the Mach numbers in the adjacent regions.

b. Program Description

Inputs to the program are the number of characteristics (N), the step size or length of the straight wall sections in the initial expansion (X1), the desired Mach number (M_1) and the test section height (H). Equations B-1 and B-2 are used to determine the wall turning angle for each step. A*/A is calculated using the isentropic relation,

$$\frac{A^*}{A} = M_1 \left[\frac{1+\frac{k-1}{2} M_1^2}{\frac{k+1}{2}} \right] - \frac{k+1}{2(k-1)}$$

The throat half-height (H1) is given by

$$H1 = \frac{H(A^*/A)}{2}$$

Referring to Figure B-1, first the coordinates of points P(1) through P(N) are calculated using the degree of turning (D) and the length (X1) for each wall section. The x coordinate of P is P(1,I) and the y coordinate is P(2,I). For the Ith point,

$$P(1,I) = P[1,(I-1)] + x1 \cos [(I-1)D]$$

and

$$P(2,I) = P[2,(I-1)] + x1 \sin [(I-1)D]$$

Succeeding points are calculated as the intersection of a left running characteristic with a right running characteristic whose angles are known and from points whose coordinates are known.

Referring to Figure B-2, when the coordinates $P_1(x_1,y_1)$ and and $P_2(x_2,y_1)$ and the angles (θ_1 and θ_2) of the characteristics are known, the coordinates of P_3 can be calculated using the equations

$$\tan \theta_1 = \frac{y_3-y_1}{x_3-x_1} \qquad \text{(B-4)}$$

and

$$\tan \theta_2 = \frac{y_3-y_2}{x_3-x_2} \qquad \text{(B-5)}$$

Combining Eqs. B-4 and B-5

$$x_3 = \frac{x_2 \tan \theta_2 - y_2 + y_1 - x_1 \tan \theta_1}{\tan \theta_2 - \tan \theta_1}$$

and

$$y_3 = (x_3 - x_1) \tan \theta_1 - y_1$$

The path of the calculations is shown in Figure B-1. Points along the left running characteristics are computed first, beginning at the point of symmetry on the nozzle centerline. At Q(1), the wall turns back through an angle (D) to cancel the characteristic. These points are calculated in a similar manner using the wall slope in place of the right running characteristic. Each left running characteristic is treated until point Q(N) is reached.

BASIC program "NOZZLE" was written for the HP-9830A calculator to carry out the above procedure for an arbitrary choice of step size and Mach number. The program listing is given in TABLE B-II.

c. Nozzle Design

Shown in TABLE B-III and Figure B-3 is the output from program "NOZZLE" for the same inputs used for the NSWC contours listed in TABLE B-I. A comparison of the nozzle designs is given in the following table:

	NSWC 12" Nozzle	NSWC Min. Length Nozzle	"NOZZLE"
Nozzle length, in	12	4.212	5.27
Percent error in area ratio	.001	.001	.021

As can be seen, "NOZZLE" provides a very rapid expansion. This is because a design constraint is that the initial expansion be complete before the first left running characteristic arrives at the wall. Accuracy in the final area ratio can be improved by increasing the number or characteristics.

U.S. NAVAL ORDANCE LABORATORY, WHITE OAK, MARYLAND

COMPUTER PROGRAM FOR THE DESIGN OF TWO-DIMENSIONAL SUPERSONIC NOZZLES

PART I

THE ISENTROPIC CORE OF THE NOZZLE

MACH 1.4 NOZZLE FOR NAVAL POST. SCHOOL 12.0 INCH LENGTH

INPUT DATA

CARD NO. 1	INDSR =	2
	INDYTH =	1
	INDXTE =	1
	INDOUT =	2
	INDCAL =	1
	INDTAP =	2
	INDCRD =	2
CARD NO. 2	GAMMA =	.14000E+01
	SMT =	.14000E+01
	XEXXT =	.12000E+02
	YTHEX =	.23640E+01
	RCT =	.50000E+02
	DELMUL =	.70000E+00

TABLE B-I. NSWC Supersonic Nozzle Program Results

NOZZLE PARAMETERS (LENGTHS IN INCHES)

MACH NO.	YTH	YEXIT	XEXIT	XT	RCT	B
.14000E+01	.21203E+01	.23640E+01	.12000E+02	.96838E+01	.50000E+02	-.17873E+00

CENTERLINE MACH NUMBER DISTRIBUTION USING CUBIC EQUATION

N	X	MACH NO.
1	.134748E-02	.100010E+01
2	.158388E-02	.100012E+01
3	.182026E-02	.100014E+01
4	.205666E-02	.100015E+01
5	.229308E-02	.100017E+01
6	.252948E-02	.100019E+01
7	.276588E-02	.100021E+01
8	.300228E-02	.100023E+01
9	.323868E-02	.100024E+01
10	.347508E-02	.100026E+01
11	.371148E-02	.100028E+01
12	.394788E-02	.100030E+01
13	.418428E-02	.100031E+01
14	.442068E-02	.100033E+01
15	.465708E-02	.100035E+01
16	.489348E-02	.100037E+01
17	.512988E-02	.100039E+01
18	.536628E-02	.100040E+01
19	.560268E-02	.100042E+01
20	.583908E-02	.100044E+01
21	.670669E-02	.100050E+01
22	.769978E-02	.100058E+01
23	.883543E-02	.100066E+01
24	.101328E-01	.100076E+01
25	.116130E-01	.100087E+01
26	.132999E-01	.100100E+01
27	.152193E-01	.100114E+01
28	.173997E-01	.100131E+01
29	.198723E-01	.100149E+01
30	.226706E-01	.100170E+01
31	.258305E-01	.100194E+01
32	.293906E-01	.100221E+01
33	.333910E-01	.100251E+01
34	.378740E-01	.100285E+01
35	.428831E-01	.100322E+01
36	.484625E-01	.100364E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

37	.546570E-01	.100410E+01
38	.615109E-01	.100462E+01
39	.690677E-01	.100518E+01
40	.773693E-01	.100580E+01
41	.864557E-01	.100648E+01
42	.963639E-01	.100722E+01
43	.107128E+00	.100803E+01
44	.118779E+00	.100890E+01
45	.131343E+00	.100983E+01
46	.144844E+00	.101084E+01
47	.159300E+00	.101191E+01
48	.174727E+00	.101306E+01
49	.191135E+00	.101428E+01
50	.208533E+00	.101557E+01
51	.226925E+00	.101693E+01
52	.246311E+00	.101836E+01
53	.266690E+00	.101987E+01
54	.288057E+00	.102144E+01
55	.310407E+00	.102309E+01
56	.333730E+00	.102480E+01
57	.358018E+00	.102658E+01
58	.383257E+00	.102843E+01
59	.409436E+00	.103034E+01
60	.436541E+00	.103231E+01
61	.464558E+00	.103435E+01
62	.493471E+00	.103645E+01
63	.523266E+00	.103860E+01
64	.553926E+00	.104082E+01
65	.585439E+00	.104309E+01
66	.617785E+00	.104541E+01
67	.650950E+00	.104779E+01
68	.684917E+00	.105022E+01
69	.719672E+00	.105269E+01
70	.755199E+00	.105522E+01
71	.791483E+00	.105779E+01
72	.828506E+00	.106040E+01
73	.866261E+00	.106306E+01
74	.904726E+00	.106576E+01
75	.943890E+00	.106850E+01
76	.983739E+00	.107128E+01
77	.102426E+01	.107409E+01
78	.106544E+01	.107695E+01
79	.110726E+01	.107983E+01
80	.114972E+01	.108275E+01
81	.119280E+01	.108570E+01
82	.123649E+01	.108866E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

83	.128077E+01	.109169E+01
84	.132564E+01	.109473E+01
85	.137109E+01	.109779E+01
86	.141710E+01	.110086E+01
87	.146367E+01	.110395E+01
88	.151079E+01	.110713E+01
89	.155843E+01	.111029E+01
90	.160661E+01	.111346E+01
91	.165530E+01	.111666E+01
92	.170450E+01	.111986E+01
93	.175420E+01	.112311E+01
94	.180439E+01	.112636E+01
95	.185506E+01	.112962E+01
96	.190620E+01	.113290E+01
97	.195782E+01	.113619E+01
98	.200989E+01	.113949E+01
99	.206241E+01	.114281E+01
100	.211536E+01	.114613E+01
101	.216879E+01	.114946E+01
102	.222263E+01	.115280E+01
103	.227689E+01	.115615E+01
104	.233157E+01	.115951E+01
105	.238667E+01	.116287E+01
106	.244216E+01	.116623E+01
107	.249806E+01	.116960E+01
108	.255436E+01	.117297E+01
109	.261104E+01	.117634E+01
110	.266810E+01	.117971E+01
111	.272554E+01	.118309E+01
112	.278336E+01	.118646E+01
113	.284154E+01	.118983E+01
114	.290008E+01	.119320E+01
115	.295896E+01	.119657E+01
116	.301824E+01	.119993E+01
117	.307784E+01	.120329E+01
118	.313779E+01	.120664E+01
119	.319807E+01	.120998E+01
120	.325869E+01	.121332E+01
121	.331964E+01	.121665E+01
122	.338092E+01	.121997E+01
123	.344252E+01	.122329E+01
124	.350443E+01	.122659E+01
125	.356667E+01	.122988E+01
126	.362921E+01	.123316E+01
127	.369206E+01	.123643E+01
128	.375521E+01	.123969E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

129	.381867E+01	.124293E+01
130	.388242E+01	.124615E+01
131	.394646E+01	.124937E+01
132	.401060E+01	.125256E+01
133	.407542E+01	.125574E+01
134	.414032E+01	.125891E+01
135	.420550E+01	.126205E+01
136	.427097E+01	.126516E+01
137	.433670E+01	.126826E+01
138	.440271E+01	.127137E+01
139	.446898E+01	.127444E+01
140	.453552E+01	.127746E+01
141	.460233E+01	.128050E+01
142	.466939E+01	.128351E+01
143	.473672E+01	.128648E+01
144	.480429E+01	.128944E+01
145	.487212E+01	.129236E+01
146	.494020E+01	.129527E+01
147	.500853E+01	.129815E+01
148	.507710E+01	.130100E+01
149	.514591E+01	.130382E+01
150	.521496E+01	.130662E+01
151	.528426E+01	.130939E+01
152	.535376E+01	.131212E+01
153	.542354E+01	.131483E+01
154	.549354E+01	.131751E+01
155	.556376E+01	.132016E+01
156	.563421E+01	.132278E+01
157	.570488E+01	.132536E+01
158	.577577E+01	.132791E+01
159	.584689E+01	.133043E+01
160	.591823E+01	.133291E+01
161	.598978E+01	.133536E+01
162	.606155E+01	.133777E+01
163	.613353E+01	.134015E+01
164	.620572E+01	.134249E+01
165	.627812E+01	.134479E+01
166	.635073E+01	.134706E+01
167	.642355E+01	.134929E+01
168	.649657E+01	.135147E+01
169	.656979E+01	.135362E+01
170	.664322E+01	.135573E+01
171	.671684E+01	.135780E+01
172	.679066E+01	.135982E+01
173	.686468E+01	.136181E+01
174	.693889E+01	.136375E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

175	.701330E+01	.136564E+01
176	.708789E+01	.136750E+01
177	.716268E+01	.136930E+01
178	.723765E+01	.137107E+01
179	.731282E+01	.137279E+01
180	.738816E+01	.137446E+01
181	.746370E+01	.137606E+01
182	.753941E+01	.137766E+01
183	.761531E+01	.137918E+01
184	.769136E+01	.138066E+01
185	.776764E+01	.138209E+01
186	.784407E+01	.138347E+01
187	.792068E+01	.138480E+01
188	.799746E+01	.138608E+01
189	.807442E+01	.138731E+01
190	.815154E+01	.138848E+01
191	.822884E+01	.138960E+01
192	.830631E+01	.139067E+01
193	.838395E+01	.139166E+01
194	.846176E+01	.139264E+01
195	.853973E+01	.139354E+01
196	.861787E+01	.139436E+01
197	.869617E+01	.139517E+01
198	.877463E+01	.139590E+01
199	.885326E+01	.139658E+01
200	.893205E+01	.139719E+01
201	.901099E+01	.139775E+01
202	.909010E+01	.139824E+01
203	.916936E+01	.139868E+01
204	.924878E+01	.139906E+01
205	.932835E+01	.139937E+01
206	.940808E+01	.139962E+01
207	.948796E+01	.139981E+01
208	.956800E+01	.139993E+01
209	.964819E+01	.139999E+01
210	.968376E+01	.140000E+01
211	.976417E+01	.140000E+01
212	.984472E+01	.140000E+01
213	.992542E+01	.140000E+01
214	.100063E+02	.140000E+01
215	.100873E+02	.140000E+01
216	.101684E+02	.140000E+01
217	.102497E+02	.140000E+01
218	.103311E+02	.140000E+01
219	.104127E+02	.140000E+01
220	.104944E+02	.140000E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

221	.105762E+02	.140000E+01
222	.106582E+02	.140000E+01
223	.107404E+02	.140000E+01
224	.108226E+02	.140000E+01
225	.109051E+02	.140000E+01
226	.109876E+02	.140000E+01
227	.110703E+02	.140000E+01
228	.111531E+02	.140000E+01
229	.112361E+02	.140000E+01
230	.113191E+02	.140000E+01
231	.114024E+02	.140000E+01
232	.114857E+02	.140000E+01
233	.115692E+02	.140000E+01
234	.116528E+02	.140000E+01
235	.117366E+02	.140000E+01
236	.118204E+02	.140000E+01
237	.119044E+02	.140000E+01
238	.119886E+02	.140000E+01
239	.120000E+02	.140000E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

CHARACTERISTICS MANIPULATION (* INTERPOLATED WALL VALUES)

	N	K	X(INCHES)	Y(INCHES)	MACH NO.	STREAM ANGLE
*	79	72	.350943E+00	.212243E+01	.104950E+01	.480875E+00
*	80	58	.377542E+00	.212266E+01	.105135E+01	.506843E+00
*	81	54	.404773E+00	.212290E+01	.105322E+01	.532752E+00
*	82	52	.432628E+00	.212317E+01	.105511E+01	.558944E+00
*	83	50	.461097E+00	.212345E+01	.105705E+01	.585398E+00
*	84	49	.490172E+00	.212376E+01	.105901E+01	.612126E+00
*	85	48	.519644E+00	.212408E+01	.106101E+01	.639147E+00
*	86	47	.550106E+00	.212443E+01	.106305E+01	.666279E+00
*	87	47	.580949E+00	.212479E+01	.106512E+01	.693598E+00
*	88	46	.612366E+00	.212518E+01	.106723E+01	.721090E+00
*	89	46	.644349E+00	.212559E+01	.106937E+01	.748734E+00
*	90	46	.676889E+00	.212602E+01	.107153E+01	.776509E+00
*	91	46	.709980E+00	.212648E+01	.107373E+01	.804392E+00
*	92	45	.743613E+00	.212696E+01	.107596E+01	.832363E+00
*	93	45	.777782E+00	.212747E+01	.107821E+01	.860401E+00
*	94	45	.812478E+00	.212800E+01	.108049E+01	.888483E+00
*	95	45	.847695E+00	.212855E+01	.108280E+01	.916592E+00
*	96	45	.883426E+00	.212913E+01	.108514E+01	.944705E+00
*	97	45	.919664E+00	.212974E+01	.108750E+01	.972804E+00
*	98	45	.956401E+00	.213037E+01	.108988E+01	.100087E+01
*	99	45	.993633E+00	.213103E+01	.109229E+01	.102888E+01
*	100	45	.103135E+01	.213172E+01	.109472E+01	.105682E+01
*	101	45	.106955E+01	.213243E+01	.109717E+01	.108468E+01
*	102	45	.110822E+01	.213317E+01	.109964E+01	.111242E+01
*	103	45	.114737E+01	.213394E+01	.110213E+01	.114004E+01
*	104	45	.118697E+01	.213474E+01	.110465E+01	.116752E+01
*	105	45	.122703E+01	.213556E+01	.110718E+01	.119484E+01
*	106	45	.126755E+01	.213642E+01	.110973E+01	.122198E+01
*	107	46	.130851E+01	.213730E+01	.111229E+01	.124894E+01
*	108	46	.134991E+01	.213821E+01	.111487E+01	.127568E+01
*	109	46	.139175E+01	.213915E+01	.111747E+01	.130221E+01
*	110	46	.143401E+01	.214012E+01	.112009E+01	.132850E+01
*	111	46	.147671E+01	.214112E+01	.112271E+01	.135453E+01
*	112	46	.151982E+01	.214215E+01	.112536E+01	.138030E+01
*	113	46	.156335E+01	.214321E+01	.112801E+01	.140580E+01
*	114	46	.160730E+01	.214430E+01	.113068E+01	.143099E+01
*	115	47	.165165E+01	.214542E+01	.113336E+01	.145589E+01
*	116	47	.169640E+01	.214656E+01	.113604E+01	.148046E+01
*	117	47	.174156E+01	.214774E+01	.113874E+01	.150470E+01
*	118	47	.178711E+01	.214895E+01	.114145E+01	.152859E+01
*	119	47	.183305E+01	.215018E+01	.114417E+01	.155213E+01
*	120	47	.187938E+01	.215145E+01	.114690E+01	.157530E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

*	121	47	.192609E+01	.215274E+01	.114963E+01	.159809E+01
*	122	48	.197319E+01	.215406E+01	.115238E+01	.162049E+01
*	123	48	.202066E+01	.215541E+01	.115512E+01	.164249E+01
*	124	48	.206851E+01	.215680E+01	.115788E+01	.166407E+01
*	125	48	.211673E+01	.215821E+01	.116064E+01	.168522E+01
*	126	48	.216532E+01	.215964E+01	.116340E+01	.170595E+01
*	127	48	.221427E+01	.216111E+01	.116617E+01	.172623E+01
*	128	48	.226359E+01	.216260E+01	.116894E+01	.174605E+01
*	129	49	.231326E+01	.216413E+01	.117171E+01	.176541E+01
*	130	49	.236330E+01	.216568E+01	.117449E+01	.178430E+01
*	131	49	.241369E+01	.216725E+01	.117727E+01	.180270E+01
*	132	49	.246443E+01	.216886E+01	.118005E+01	.182061E+01
*	133	49	.251552E+01	.217049E+01	.118283E+01	.183802E+01
*	134	49	.256697E+01	.217215E+01	.118561E+01	.185493E+01
*	135	50	.261875E+01	.217383E+01	.118839E+01	.187131E+01
*	136	50	.267089E+01	.217554E+01	.119117E+01	.188717E+01
*	137	50	.272336E+01	.217726E+01	.119394E+01	.190249E+01
*	138	50	.277618E+01	.217904E+01	.119672E+01	.191728E+01
*	139	50	.282934E+01	.218083E+01	.119949E+01	.193151E+01
*	140	50	.288283E+01	.218264E+01	.120226E+01	.194518E+01
*	141	51	.293666E+01	.218447E+01	.120502E+01	.195829E+01
*	142	51	.299083E+01	.218633E+01	.120779E+01	.197083E+01
*	143	51	.304533E+01	.218821E+01	.121054E+01	.198279E+01
*	144	51	.310016E+01	.219011E+01	.121329E+01	.199416E+01
*	145	51	.315533E+01	.219204E+01	.121604E+01	.200494E+01
*	146	51	.321082E+01	.219399E+01	.121878E+01	.201512E+01
*	147	52	.326665E+01	.219596E+01	.122151E+01	.202469E+01
*	148	52	.332280E+01	.219794E+01	.122423E+01	.203365E+01
*	149	52	.337928E+01	.219995E+01	.122695E+01	.204199E+01
*	150	52	.343609E+01	.220198E+01	.122966E+01	.204970E+01
*	151	52	.349323E+01	.220403E+01	.123235E+01	.205678E+01
*	152	52	.355069E+01	.220610E+01	.123504E+01	.206323E+01
*	153	53	.360848E+01	.220818E+01	.123772E+01	.206903E+01
*	154	53	.366660E+01	.221029E+01	.124039E+01	.207418E+01
*	155	53	.372504E+01	.221240E+01	.124305E+01	.207868E+01
*	156	53	.378380E+01	.221454E+01	.124570E+01	.208251E+01
*	157	53	.384290E+01	.221669E+01	.124834E+01	.208568E+01
*	158	53	.390231E+01	.221885E+01	.125096E+01	.208818E+01
*	159	53	.396206E+01	.222103E+01	.125357E+01	.209001E+01
*	160	54	.402212E+01	.222323E+01	.125617E+01	.209115E+01
*	161	54	.408252E+01	.222543E+01	.125875E+01	.209161E+01
*	162	54	.414323E+01	.222765E+01	.126132E+01	.209137E+01
*	163	54	.420428E+01	.222988E+01	.126387E+01	.209044E+01
*	164	54	.426565E+01	.223212E+01	.126641E+01	.208881E+01
*	165	54	.432735E+01	.223437E+01	.126894E+01	.208648E+01
*	166	54	.438936E+01	.223662E+01	.127145E+01	.208344E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

*	167	55	.445173E+01	.223889E+01	.127394E+01	.207960E+01
*	168	55	.451442E+01	.224116E+01	.127641E+01	.207520E+01
*	169	55	.457743E+01	.224344E+01	.127887E+01	.207001E+01
*	170	55	.464077E+01	.224573E+01	.128131E+01	.206409E+01
*	171	55	.470445E+01	.224802E+01	.128373E+01	.205743E+01
*	172	55	.476846E+01	.225032E+01	.128613E+01	.205005E+01
*	173	55	.483280E+01	.225262E+01	.128851E+01	.204193E+01
*	174	56	.489747E+01	.225492E+01	.129087E+01	.203307E+01
*	175	56	.496249E+01	.225722E+01	.129321E+01	.202346E+01
*	176	56	.502783E+01	.225952E+01	.129553E+01	.201311E+01
*	177	56	.509352E+01	.226183E+01	.129783E+01	.200201E+01
*	178	56	.515954E+01	.226413E+01	.130011E+01	.199015E+01
*	179	56	.522591E+01	.226643E+01	.130236E+01	.197754E+01
*	180	56	.529262E+01	.226872E+01	.130459E+01	.196417E+01
*	181	56	.535967E+01	.227101E+01	.130680E+01	.195004E+01
*	182	56	.542706E+01	.227330E+01	.130898E+01	.193515E+01
*	183	57	.549480E+01	.227558E+01	.131114E+01	.191949E+01
*	184	57	.556289E+01	.227785E+01	.131328E+01	.190306E+01
*	185	57	.563133E+01	.228011E+01	.131539E+01	.188586E+01
*	186	57	.570012E+01	.228237E+01	.131747E+01	.186788E+01
*	187	57	.576926E+01	.228461E+01	.131953E+01	.184914E+01
*	188	57	.583876E+01	.228684E+01	.132156E+01	.182961E+01
*	189	57	.590861E+01	.228906E+01	.132356E+01	.180931E+01
*	190	57	.597882E+01	.229127E+01	.132553E+01	.178824E+01
*	191	57	.604940E+01	.229346E+01	.132746E+01	.176638E+01
*	192	57	.612033E+01	.229563E+01	.132940E+01	.174374E+01
*	193	58	.619163E+01	.229779E+01	.133128E+01	.172032E+01
*	194	58	.626330E+01	.229992E+01	.133314E+01	.169612E+01
*	195	58	.633533E+01	.230204E+01	.133497E+01	.167113E+01
*	196	58	.640773E+01	.230414E+01	.133676E+01	.164537E+01
*	197	58	.648051E+01	.230621E+01	.133853E+01	.161882E+01
*	198	58	.655366E+01	.230826E+01	.134026E+01	.159149E+01
*	199	58	.662719E+01	.231029E+01	.134196E+01	.156337E+01
*	200	58	.670109E+01	.231229E+01	.134362E+01	.153448E+01
*	201	58	.677538E+01	.231426E+01	.134525E+01	.150481E+01
*	202	58	.685005E+01	.231620E+01	.134685E+01	.147436E+01

TABLE B-I. NSWC Supersonic Nozzle Program Results

*	203	58	.692511E+01	.231811E+01	.134841E+01	.144313E+01
*	204	56	.700055E+01	.231999E+01	.134993E+01	.141112E+01
*	205	58	.707639E+01	.232184E+01	.135142E+01	.137835E+01
*	206	58	.715262E+01	.232365E+01	.135287E+01	.134480E+01
*	207	58	.722924E+01	.232542E+01	.135428E+01	.131049E+01
*	208	58	.730626E+01	.232716E+01	.135565E+01	.127540E+01
*	209	58	.738369E+01	.232886E+01	.135699E+01	.123956E+01
*	210	59	.746150E+01	.233052E+01	.135829E+01	.120296E+01
*	211	58	.753971E+01	.233214E+01	.135954E+01	.116560E+01
*	212	57	.761832E+01	.233372E+01	.136076E+01	.112749E+01
*	213	56	.769733E+01	.233526E+01	.136193E+01	.108863E+01
*	214	55	.777675E+01	.233676E+01	.136306E+01	.104902E+01
*	215	54	.785658E+01	.233822E+01	.136414E+01	.100867E+01
*	216	53	.793682E+01	.233964E+01	.136518E+01	.967581E+00
*	217	52	.801747E+01	.234102E+01	.136617E+01	.925760E+00
*	218	51	.809854E+01	.234236E+01	.136712E+01	.883210E+00
*	219	50	.818003E+01	.234366E+01	.136802E+01	.839933E+00
*	220	49	.826194E+01	.234492E+01	.136887E+01	.795933E+00
*	221	48	.834428E+01	.234614E+01	.136967E+01	.751214E+00
*	222	47	.842705E+01	.234732E+01	.137042E+01	.705780E+00
*	223	46	.851025E+01	.234846E+01	.137112E+01	.659635E+00
*	224	45	.859389E+01	.234956E+01	.137177E+01	.612784E+00
*	225	44	.867797E+01	.235062E+01	.137236E+01	.565232E+00
*	226	43	.876250E+01	.235163E+01	.137290E+01	.516984E+00
*	227	42	.884748E+01	.235261E+01	.137339E+01	.468046E+00
*	228	41	.893291E+01	.235354E+01	.137382E+01	.418423E+00
*	229	40	.901881E+01	.235443E+01	.137419E+01	.368121E+00
*	230	39	.910517E+01	.235527E+01	.137451E+01	.317147E+00
*	231	38	.919200E+01	.235607E+01	.137477E+01	.265508E+00
*	232	37	.927931E+01	.235682E+01	.137497E+01	.213211E+00
*	233	36	.936710E+01	.235752E+01	.137511E+01	.160264E+00
*	234	35	.945538E+01	.235817E+01	.137519E+01	.106675E+00
*	235	34	.954416E+01	.235877E+01	.137521E+01	.524523E-01
*	236	33	.963344E+01	.235932E+01	.137517E+01	.-.284000E-02
*	237	31	.972324E+01	.235982E+01	.137506E+01	.-.583000E-01
*	238	31	.981356E+01	.236026E+01	.137490E+01	.-.113000E+00

TABLE B-I. NSWC Supersonic Nozzle Program Results

FINAL OUTPUT TO PART I, ISENTROPIC CORE CONTOUR

I	X(INCHES)	Y(INCHES)	CL MACH NO.	WALL MACH NO.	WALL ANGLE	RAD OF CURV (IN)
1	.377542E+00	.212266E+01	.102801E+01	.105135E+01	.506843E+00	0.
2	.404773E+00	.212290E+01	.103000E+01	.105322E+01	.532752E+00	.605859E+02
3	.432628E+00	.212317E+01	.103203E+01	.105511E+01	.558944E+00	.613057E+02
4	.461097E+00	.212345E+01	.103410E+01	.105705E+01	.585398E+00	.620000E+02
5	.490172E+00	.212376E+01	.103621E+01	.105901E+01	.612128E+00	.626325E+02
6	.519844E+00	.212408E+01	.103836E+01	.106101E+01	.639147E+00	.634238E+02
7	.550106E+00	.212443E+01	.104054E+01	.106305E+01	.666279E+00	.643066E+02
8	.580949E+00	.212479E+01	.104276E+01	.106512E+01	.693598E+00	.650920E+02
9	.612366E+00	.212518E+01	.104502E+01	.106723E+01	.721090E+00	.658920E+02
10	.644349E+00	.212559E+01	.104732E+01	.106937E+01	.748734E+00	.667187E+02
11	.676889E+00	.212602E+01	.104964E+01	.107153E+01	.776509E+00	.675737E+02
12	.709980E+00	.212648E+01	.105200E+01	.107373E+01	.804392E+00	.684580E+02
13	.743613E+00	.212696E+01	.105439E+01	.107596E+01	.832363E+00	.693731E+02
14	.777782E+00	.212747E+01	.105682E+01	.107821E+01	.860401E+00	.703206E+02
15	.812478E+00	.212800E+01	.105927E+01	.108049E+01	.888483E+00	.713017E+02
16	.847695E+00	.212855E+01	.106175E+01	.108280E+01	.916592E+00	.723180E+02
17	.883426E+00	.212913E+01	.106427E+01	.108514E+01	.944705E+00	.733708E+02
18	.919664E+00	.212974E+01	.106681E+01	.108750E+01	.972804E+00	.744618E+02
19	.956401E+00	.213037E+01	.106937E+01	.108988E+01	.100087E+01	.755926E+02
20	.993633E+00	.213103E+01	.107197E+01	.109229E+01	.102888E+01	.767651E+02
21	.103135E+01	.213172E+01	.107459E+01	.109472E+01	.105682E+01	.779812E+02
22	.106955E+01	.213243E+01	.107723E+01	.109717E+01	.108468E+01	.792429E+02
23	.110822E+01	.213317E+01	.107990E+01	.109964E+01	.111242E+01	.805523E+02
24	.114737E+01	.213394E+01	.108259E+01	.110213E+01	.114004E+01	.819118E+02
25	.118697E+01	.213474E+01	.108530E+01	.110465E+01	.116752E+01	.833238E+02
26	.122703E+01	.213556E+01	.108804E+01	.110718E+01	.119484E+01	.847911E+02
27	.126755E+01	.213642E+01	.109079E+01	.110973E+01	.122198E+01	.863163E+02
28	.130851E+01	.213730E+01	.109357E+01	.111229E+01	.124894E+01	.879025E+02
29	.134991E+01	.213821E+01	.109637E+01	.111487E+01	.127568E+01	.895533E+02
30	.139175E+01	.213915E+01	.109918E+01	.111747E+01	.130221E+01	.912723E+02
31	.143401E+01	.214012E+01	.110201E+01	.112009E+01	.132850E+01	.930631E+02
32	.147671E+01	.214112E+01	.110486E+01	.112271E+01	.135453E+01	.949300E+02
33	.151982E+01	.214215E+01	.110773E+01	.112536E+01	.138030E+01	.968775E+02
34	.156335E+01	.214321E+01	.111061E+01	.112801E+01	.140580E+01	.989105E+02
35	.160730E+01	.214430E+01	.111351E+01	.113068E+01	.143099E+01	.101034E+03
36	.165165E+01	.214542E+01	.111642E+01	.113336E+01	.145589E+01	.103254E+03
37	.169640E+01	.214656E+01	.111935E+01	.113604E+01	.148046E+01	.105578E+03
38	.174156E+01	.214774E+01	.112229E+01	.113874E+01	.150470E+01	.108011E+03
39	.178711E+01	.214895E+01	.112524E+01	.114145E+01	.152859E+01	.110561E+03
40	.183305E+01	.215018E+01	.112821E+01	.114417E+01	.155213E+01	.113237E+03
41	.187938E+01	.215145E+01	.113118E+01	.114690E+01	.157530E+01	.116047E+03
42	.192609E+01	.215274E+01	.113417E+01	.114963E+01	.159809E+01	.119002E+03

TABLE B-I. NSWC Supersonic Nozzle Program Results

43	.197319E+01	.215406E+01	.113717E+01	.115238E+01	.162049E+01	.122113E+03
44	.202066E+01	.215541E+01	.114017E+01	.115512E+01	.164249E+01	.125391E+03
45	.206851E+01	.215680E+01	.114319E+01	.115788E+01	.166407E+01	.128851E+03
46	.211673E+01	.215821E+01	.114621E+01	.116064E+01	.168522E+01	.132506E+03
47	.216532E+01	.215964E+01	.114925E+01	.116340E+01	.170595E+01	.136374E+03
48	.221427E+01	.216111E+01	.115229E+01	.116617E+01	.172623E+01	.140474E+03
49	.226359E+01	.216260E+01	.115533E+01	.116894E+01	.174605E+01	.144826E+03
50	.231326E+01	.216413E+01	.115839E+01	.117171E+01	.176541E+01	.149455E+03
51	.236330E+01	.216568E+01	.116144E+01	.117449E+01	.178430E+01	.154385E+03
52	.241369E+01	.216725E+01	.116451E+01	.117727E+01	.180270E+01	.159647E+03
53	.246443E+01	.216886E+01	.116757E+01	.118005E+01	.182061E+01	.165276E+03
54	.251552E+01	.217049E+01	.117065E+01	.118283E+01	.183802E+01	.171310E+03
55	.256697E+01	.217215E+01	.117372E+01	.118561E+01	.185493E+01	.177795E+03
56	.261875E+01	.217383E+01	.117680E+01	.118839E+01	.187131E+01	.184783E+03
57	.267089E+01	.217554E+01	.117988E+01	.119117E+01	.188717E+01	.192331E+03
58	.272336E+01	.217728E+01	.118296E+01	.119394E+01	.190249E+01	.200511E+03
59	.277618E+01	.217904E+01	.118604E+01	.119672E+01	.191728E+01	.209403E+03
60	.282934E+01	.218083E+01	.118913E+01	.119949E+01	.193151E+01	.219105E+03
61	.288283E+01	.218264E+01	.119221E+01	.120226E+01	.194518E+01	.229732E+03
62	.293666E+01	.218447E+01	.119529E+01	.120502E+01	.195829E+01	.241425E+03
63	.299083E+01	.218633E+01	.119838E+01	.120779E+01	.197083E+01	.254344E+03
64	.304533E+01	.218821E+01	.120146E+01	.121054E+01	.198279E+01	.268694E+03
65	.310016E+01	.219011E+01	.120454E+01	.121329E+01	.199416E+01	.284724E+03
66	.315533E+01	.219204E+01	.120761E+01	.121604E+01	.200494E+01	.302748E+03
67	.321082E+01	.219399E+01	.121069E+01	.121878E+01	.201512E+01	.323162E+03
68	.326665E+01	.219596E+01	.121376E+01	.122151E+01	.202469E+01	.346479E+03
69	.332280E+01	.219794E+01	.121682E+01	.122423E+01	.203365E+01	.373351E+03
70	.337928E+01	.219995E+01	.121989E+01	.122695E+01	.204199E+01	.404649E+03
71	.343609E+01	.220198E+01	.122294E+01	.122966E+01	.204970E+01	.441569E+03
72	.349323E+01	.220403E+01	.122599E+01	.123235E+01	.205678E+01	.485770E+03
73	.355069E+01	.220610E+01	.122904E+01	.123504E+01	.206323E+01	.539639E+03
74	.360848E+01	.220818E+01	.123208E+01	.123772E+01	.206903E+01	.606761E+03
75	.366660E+01	.221029E+01	.123511E+01	.124039E+01	.207418E+01	.692668E+03
76	.372504E+01	.221240E+01	.123813E+01	.124305E+01	.207868E+01	.806453E+03
77	.378380E+01	.221454E+01	.124115E+01	.124570E+01	.208251E+01	.964358E+03
78	.384290E+01	.221669E+01	.124416E+01	.124834E+01	.208568E+01	.119818E+04
79	.390231E+01	.221885E+01	.124716E+01	.125096E+01	.208818E+01	.157995E+04
80	.396206E+01	.222103E+01	.125014E+01	.125357E+01	.209001E+01	.231524E+04
81	.402212E+01	.222323E+01	.125312E+01	.125617E+01	.209115E+01	.431826E+04
82	.408252E+01	.222543E+01	.125609E+01	.125875E+01	.209161E+01	.312331E+05
83	.414323E+01	.222765E+01	.125905E+01	.126132E+01	.209137E+01	-.599800E+04
84	.420428E+01	.222988E+01	.126199E+01	.126387E+01	.209044E+01	-.274227E+04
85	.426565E+01	.223212E+01	.126492E+01	.126641F+01	.208881E+01	-.178006E+04
86	.432735E+01	.223437E+01	.126784E+01	.126894E+01	.208648E+01	-.131918E+04
87	.438938E+01	.223662E+01	.127075E+01	.127145F+01	.208344E+01	-.104873E+04
88	.445173E+01	.223889E+01	.127364E+01	.127394E+01	.207968E+01	-.870941E+03

TABLE B-I. NSWC Supersonic Nozzle Program Results

89	.451442E+01	.224116E+01	.127652E+01	.127641E+01	.207520E+01	-.745224E+03
90	.457743E+01	.224344E+01	.127938E+01	.127667E+01	.207001E+01	-.651634E+03
91	.464077E+01	.224573E+01	.128223E+01	.128131E+01	.206409E+01	-.579250E+03
92	.470445E+01	.224802E+01	.128506E+01	.128373E+01	.205743E+01	-.521608E+03
93	.476846E+01	.225032E+01	.128787E+01	.128613E+01	.205005E+01	-.474627E+03
94	.483280E+01	.225262E+01	.129067E+01	.128851E+01	.204193E+01	-.435602E+03
95	.489747E+01	.225492E+01	.129345E+01	.129087E+01	.203307E+01	-.402664E+03
96	.496249E+01	.225722E+01	.129621E+01	.129321E+01	.202346E+01	-.374508E+03
97	.502783E+01	.225952E+01	.129895E+01	.129553E+01	.201311E+01	-.350179E+03
98	.509352E+01	.226183E+01	.130167E+01	.129783E+01	.200201E+01	-.328941E+03
99	.515954E+01	.226413E+01	.130438E+01	.130011E+01	.199015E+01	-.310244E+03
100	.522591E+01	.226643E+01	.130706E+01	.130236E+01	.197754E+01	-.293662E+03
101	.529262E+01	.226872E+01	.130972E+01	.130459E+01	.196417E+01	-.278859E+03
102	.535967E+01	.227101E+01	.131235E+01	.130680E+01	.195004E+01	-.265564E+03
103	.542706E+01	.227330E+01	.131497E+01	.130898E+01	.193515E+01	-.253558E+03
104	.549480E+01	.227558E+01	.131756E+01	.131114E+01	.191949E+01	-.242665E+03
105	.556289E+01	.227785E+01	.132013E+01	.131328E+01	.190306E+01	-.232746E+03
106	.563133E+01	.228011E+01	.132267E+01	.131539E+01	.188586E+01	-.223677E+03
107	.570012E+01	.228237E+01	.132519E+01	.131747E+01	.186788E+01	-.215353E+03
108	.576926E+01	.228461E+01	.132768E+01	.131953E+01	.184914E+01	-.207689E+03
109	.583876E+01	.228684E+01	.133014E+01	.132156E+01	.182961E+01	-.200612E+03
110	.590861E+01	.228906E+01	.133258E+01	.132356E+01	.180931E+01	-.194058E+03
111	.597882E+01	.229127E+01	.133499E+01	.132553E+01	.178824E+01	-.187973E+03
112	.604940E+01	.229346E+01	.133737E+01	.132748E+01	.176638E+01	-.182311E+03
113	.612033E+01	.229563E+01	.133972E+01	.132940E+01	.174374E+01	-.177031E+03
114	.619163E+01	.229779E+01	.134204E+01	.133128E+01	.172032E+01	-.172095E+03
115	.626330E+01	.229992E+01	.134433E+01	.133314E+01	.169612E+01	-.167475E+03
116	.633533E+01	.230204E+01	.134658E+01	.133497E+01	.167113E+01	-.163145E+03
117	.640773E+01	.230414E+01	.134881E+01	.133676E+01	.164537E+01	-.159077E+03
118	.648051E+01	.230621E+01	.135100E+01	.133853E+01	.161882E+01	-.155252E+03
119	.655366E+01	.230826E+01	.135315E+01	.134026E+01	.159149E+01	-.151649E+03
120	.662719E+01	.231029E+01	.135527E+01	.134196E+01	.156337E+01	-.148251E+03
121	.670109E+01	.231229E+01	.135736E+01	.134362E+01	.153448E+01	-.145043E+03
122	.677538E+01	.231426E+01	.135941E+01	.134525E+01	.150481E+01	-.142011E+03
123	.685005E+01	.231620E+01	.136142E+01	.134685E+01	.147436E+01	-.139143E+03
124	.692511E+01	.231811E+01	.136339E+01	.134841E+01	.144313E+01	-.136427E+03
125	.700055E+01	.231999E+01	.136532E+01	.134993E+01	.141112E+01	-.133853E+03
126	.707639E+01	.232184E+01	.136721E+01	.135142E+01	.137835E+01	-.131413E+03
127	.715262E+01	.232365E+01	.136906E+01	.135287E+01	.134480E+01	-.129096E+03
128	.722924E+01	.232542E+01	.137087E+01	.135428E+01	.131049E+01	-.126897E+03
129	.730626E+01	.232716E+01	.137264E+01	.135565E+01	.127540E+01	-.124808E+03
130	.738369E+01	.232886E+01	.137436E+01	.135699E+01	.123956E+01	-.123393E+03
131	.741810E+01	.232960E+01	.137511E+01	.135757E+01	.122346E+01	-.122722E+03
132	.757406E+01	.233283E+01	.137836E+01	.136012E+01	.115065E+01	-.124079E+03
133	.773061E+01	.233587E+01	.138141E+01	.136262E+01	.107911E+01	-.126875E+03
134	.788775E+01	.233872E+01	.138424E+01	.136507E+01	.100896E+01	-.129986E+03

TABLE B-I. NSWC Supersonic Nozzle Program Results

135	.804547E+01	.234140E+01	.138685E+01	.136746E+01	.940294E+00	-.133414E+03
136	.820376E+01	.234390E+01	.138924E+01	.136980E+01	.873218E+00	-.137192E+03
137	.836261E+01	.234622E+01	.139141E+01	.137208E+01	.807824E+00	-.141364E+03
138	.852201E+01	.234838E+01	.139334E+01	.137429E+01	.744208E+00	-.145985E+03
139	.868195E+01	.235036E+01	.139503E+01	.137643E+01	.682470E+00	-.151118E+03
140	.884242E+01	.235218E+01	.139649E+01	.137850E+01	.622706E+00	-.156841E+03
141	.900342E+01	.235385E+01	.139770E+01	.138050E+01	.565016E+00	-.163250E+03
142	.916493E+01	.235536E+01	.139866E+01	.138242E+01	.509501E+00	-.170463E+03
143	.932695E+01	.235672E+01	.139936E+01	.138427E+01	.456261E+00	-.178627E+03
144	.948945E+01	.235794E+01	.139981E+01	.138603E+01	.405399E+00	-.187931E+03
145	.965243E+01	.235902E+01	.139999E+01	.138770E+01	.357017E+00	-.198614E+03
146	.981588E+01	.235997E+01	.140000E+01	.138928E+01	.311220E+00	-.210991E+03
147	.997979E+01	.236080E+01	.140000E+01	.139077E+01	.268114E+00	-.225482E+03
148	.101441E+02	.236150E+01	.140000E+01	.139216E+01	.227802E+00	-.242658E+03
149	.103089E+02	.236210E+01	.140000E+01	.139345E+01	.190394E+00	-.263317E+03
150	.104741E+02	.236260E+01	.140000E+01	.139463E+01	.155997E+00	-.288609E+03
151	.106397E+02	.236301E+01	.140000E+01	.139571E+01	.124719E+00	-.320257E+03
152	.108057E+02	.236332E+01	.140000E+01	.139667E+01	.966701E-01	-.360958E+03
153	.109721E+02	.236357E+01	.140000E+01	.139752E+01	.719609E-01	-.415186E+03
154	.111388E+02	.236375E+01	.140000E+01	.139826E+01	.507022E-01	-.490953E+03
155	.113058E+02	.236387E+01	.140000E+01	.139887E+01	.330054E-01	-.604152E+03
156	.114732E+02	.236394E+01	.140000E+01	.139935E+01	.189824E-01	-.791415E+03
157	.116409E+02	.236398E+01	.140000E+01	.139970E+01	.874516E-02	-.116023E+04
158	.118089E+02	.236400E+01	.140000E+01	.139992E+01	.240438E-02	-.228145E+04
159	.119846E+02	.236400E+01	.140000E+01	.140000E+01	.112214E-03	-.455430E+04
160	.120000E+02	.236400E+01	.140000E+01	.140000E+01	0.	0.

TABLE B-I. NSWC Supersonic Nozzle Program Results

```
10 REM******NOZZLE******N.J.DEMO******14 JAN 1978******
20 REM
30 REM    THIS PROGRAM CAN BE USED TO CALCULATE THE
40 REM COORDINATES FOR A SUPERSONIC NOZZLE TO GIVE A
50 REM UNIFORM EXIT FLOW. THE METHOD OF CHARACTERISTICS
60 REM IS THE SOLUTION TECHNIQUE USED WITH THE SONIC
70 REM LINE ASSUMED TO BE STRAIGHT AT THE THROAT. THE
80 REM REQUIRED INPUTS ARE THE NUMBER OF CHARACTERISTICS,
90 REM STEP SIZE FOR THE INITIAL EXPANSION, DESIRED
100 REM MACH NUMBER AND THE NOZZLE EXIT HEIGHT.
110 REM
120 DISP "INPUT NO. OF CHARACTERISTICS";
130 INPUT N
140 DIM P[3,30],Q[3,30],X[30,30],Y[30,30],A[30],R[3,30]
150 DISP "INPUT STEP SIZE";
160 INPUT X1
170 DISP "INPUT DESIRED MACH NO.";
180 INPUT M1
190 DISP "INPUT TEST SECTION HEIGHT";
200 INPUT H
210 DISP "INPUT 0 FOR ANGLES, 1 FOR COORDS";
220 INPUT Z1
230 Z2=57.29577951
240 K=1.4
250 REM CALCULATE AREA RATIO----------------------------------
260 R1=M1*((1+(K-1)/2*M1↑2)/((K+1)/2))↑(-(K+1)/(2*(K-1)))
270 H1=R1*H/2
280 B=(K+1)/(K-1)
290 REM CALCULATE TOTAL TURNING-------------------------------
300 U=SQR(B)*ATN(SQR(1/B*(M1↑2-1)))-ATN(SQR(M1↑2-1))
310 REM CALCULATE ANGLE STEPS FOR WALL------------------------
320 D=U/(2*N)
330 REM BEGIN FIRST STEP--------------------------------------
340 A[1]=PI/2
350 I=2
```

TABLE B-II. "NOZZLE" Program Listing

```
360 T=(I-1)*D
370 GOSUB 1780
380 A[I]=A
390 P[3,(I-1)]=(A[I-1]-(I-2)*D+A[I]-(I-1)*D)/2
400 T=T+D
410 GOSUB 1780
420 A[I+1]=A
430 R[3,(I-1)]=(A[I]+(I-1)*D+A[I+1])/2
440 FOR I=3 TO (N+1)
450 P[3,(I-1)]=(A[I-1]-(I-2)*D+A[I]-(I-1)*D)/2
460 T=T+D
470 GOSUB 1780
480 A[I+1]=A
490 R[3,(I-1)]=(A[I]+(I-1)*D+A[I+1]+(I-2)*D)/2
500 NEXT I
510 P[1,1]=0
520 P[2,1]=H1
530 X[1,1]=H1/TAN(P[3,1])
540 Y[1,1]=0
550 FOR I=2 TO N
560 P[1,I]=P[1,(I-1)]+X1*COS((I-1)*D)
570 P[2,I]=P[2,(I-1)]+X1*SIN((I-1)*D)
580 X2=-Y[1,(I-1)]+TAN(R[3,(I-1)])*X[1,(I-1)]
590 X3=P[2,I]+TAN(P[3,I])*P[1,I]
600 X[1,I]=(X2+X3)/(TAN(P[3,I])+TAN(R[3,(I-1)]))
610 Y[1,I]=Y[1,(I-1)]+(X[1,I]-X[1,(I-1)])*TAN(R[3,(I-1)])
620 NEXT I
630 Q2=Y[1,N]-TAN(R[3,N])*X[1,N]-P[2,N]+TAN(N*D)*P[1,N]
640 Q[1,1]=Q2/(TAN(N*D)-TAN(R[3,N]))
650 Q[2,1]=Y[1,N]+(Q[1,1]-X[1,N])*TAN(R[3,N])
660 IF Q[1,1]>P[1,N] THEN 690
670 DISP "STEP SIZE TO LARGE, TRY AGAIN";
680 STOP
690 PRINT "NO. OF CHARACTERISTICS="N
700 PRINT "INPUT STEP SIZE="X1"INCHES"
```

TABLE B-II. "NOZZLE" Program Listing (Continued)

```
710 PRINT "DESIRED MACH NO.="M1
720 PRINT "TEST SECTION HEIGHT="H"INCHES"
730 IF Z1=1 THEN 800
740 PRINT "ANGLES FOR FIRST STEP"
750 REM BEGIN ITERATIONS 2 THRU N--------------------------
760 FOR I=1 TO N
770 PRINT "P(3,"I")="P[3,I]*Z2"R(3,"I")="R[3,I]*Z2
780 NEXT I
790 GOTO 840
800 FOR I=1 TO N
810 PRINT "P[1,"I")="P[1,I]"P(2,"I")="P[2,I]
820 NEXT I
830 PRINT "Q[1,1)="Q[1,1]"Q(2,1)="Q[2,1]
840 I=1
850 FOR J=1 TO (N-1)
860 T=2*J*D
870 GOSUB 1780
880 A[J]=A
890 I=J+1
900 T=(I+J)*D
910 GOSUB 1780
920 A[I]=A
930 P[3,I]=(A[I-1]+A[I]-(I-J)*D)/2
940 T=T+D
950 GOSUB 1780
960 A[I+1]=A
970 R[3,I]=(A[I]+(I-J)*D+A[I+1])/2
980 IF Z1=1 THEN 1010
990 PRINT "NEXT SET OF ANGLES"
1000 PRINT "P(3,"(J+1)")="P[3,(J+1)]*Z2"R(3,"(J+1)")="R[3,(J+1)]*Z2
1010 FOR I=(J+2) TO N
1020 P[3,I]=(A[I-1]+A[I]-(2*(I-J)-1)*D)/2
1030 T=T+D
1040 GOSUB 1780
1050 A[I+1]=A
```

TABLE B-II. "NOZZLE" Program Listing (Continued)

```
1060 R[3,I]=(A[I]+(I-J)*D+A[I+1]+(I-J-1)*D)/2
1070 IF Z1=1 THEN 1090
1080 PRINT "P(3,"I")="P[3,I]*Z2"R(3,"I")="R[3,I]*Z2
1090 NEXT I
1100 X[(J+1),(J+1)]=X[J,(J+1)]+Y[J,(J+1)]/TAN(P[3,(J+1)])
1110 Y[(J+1),(J+1)]=0
1120 FOR I=(J+2) TO N
1130 X2=Y[J,I]+X[J,I]*TAN(P[3,I])
1140 X3=X[(J+1),(I-1)]*TAN(R[3,(I-1)])-Y[(J+1),(I-1)]
1150 X[(J+1),I]=(X2+X3)/(TAN(R[3,(I-1)])+TAN(P[3,I]))
1160 Y2=(X[(J+1),I]-X[(J+1),(I-1)])*TAN(R[3,(I-1)])
1170 Y[(J+1),I]=Y2+Y[(J+1),(I-1)]
1180 NEXT I
1190 Q2=Y[(J+1),N]-TAN(R[3,N])*X[(J+1),N]-Q[2,J]+TAN((N-J)*D)*Q[1,J]
1200 Q[1,(J+1)]=Q2/(TAN((N-J)*D)-TAN(R[3,N]))
1210 Q[2,(J+1)]=Y[(J+1),N]+(Q[1,(J+1)]-X[(J+1),N])*TAN(R[3,N])
1220 IF Z1=0 THEN 1240
1230 PRINT "Q(1,"(J+1)")="Q[1,(J+1)]"Q(2,"(J+1)")="Q[2,(J+1)]
1240 NEXT J
1250 R1=1/R1
1260 R=Q[2,N]/P[2,1]
1270 E=(R1-R)/R1*100
1280 PRINT "A/A*="R
1290 PRINT "PERCENT ERROR IN A/A*="E
1300 DISP "<CONT> IF PLOT DESIRED";
1310 STOP
1320 REM THIS SECTION PRODUCES THE PLOT----------------
1330 A=Q[1,N]
1340 B=2*Q[2,N]
1350 SCALE 0,A,0,B
1360 XAXIS 0,A,0,A
1370 YAXIS 0,B,0,B
1380 PEN
1390 FOR I=1 TO N
1400 PLOT P[1,I],P[2,I],-2
```

TABLE B-II. "NOZZLE" Program Listing (Continued)

```
1410 FOR J=1 TO I
1420 PLOT X[J,I],Y[J,I]
1430 NEXT J
1440 PEN
1450 NEXT I
1460 FOR I=1 TO N
1470 PLOT X[I,I],Y[I,I],-2
1480 FOR J=(I+1) TO N
1490 PLOT X[I,J],Y[I,J]
1500 NEXT J
1510 PLOT Q[1,I],Q[2,I]
1520 PEN
1530 NEXT I
1540 PLOT P[1,1],P[2,1],-2
1550 FOR I=2 TO N
1560 PLOT P[1,I],P[2,I]
1570 NEXT I
1580 FOR I=1 TO N
1590 PLOT Q[1,I],Q[2,I]
1600 NEXT I
1610 Z=Q[1,N]*0.1
1620 PLOT Z,(0.97*B),1
1630 LABEL (*,2,2,0,0.7)"NO. OF CHARACTERISTICS="N
1640 PLOT Z,(0.94*B),1
1650 LABEL (*)"STEP SIZE="X1"INCHES"
1660 PLOT Z,(0.91*B),1
1670 LABEL (*)"MACH NO.="M
1680 PLOT Z,(0.88*B),1
1690 LABEL (*)"TEST SECTION HEIGHT="H"INCHES"
1700 PLOT Z,(0.85*B),1
1710 LABEL (*)"NOZZLE LENGTH="Q[1,N]"INCHES"
1720 PLOT Z,(0.82*B),1
1730 LABEL (*)"AREA RATIO="R
1740 STOP
1750 END
```

TABLE B-II. "NOZZLE" Program Listing (Continued)

```
1760 REM SUBROUTINE TO CALCULATE MACH NUMBER ---------------
1770 REM AND MACH ANGLE FROM A GIVEN TURNING
1780 M=1.05
1790 F=SQR(B)*ATN(SQR(1/B*(M↑2-1)))-ATN(SQR(M↑2-1))-T
1800 F1=SQR(M↑2-1)/(M*(1+(K-1)/2*M↑2))
1810 M=M-F/F1
1820 IF ABS(F)<1E-08 THEN 1840
1830 GOTO 1790
1840 A=ATN(1/SQR(M↑2-1))
1850 RETURN
```

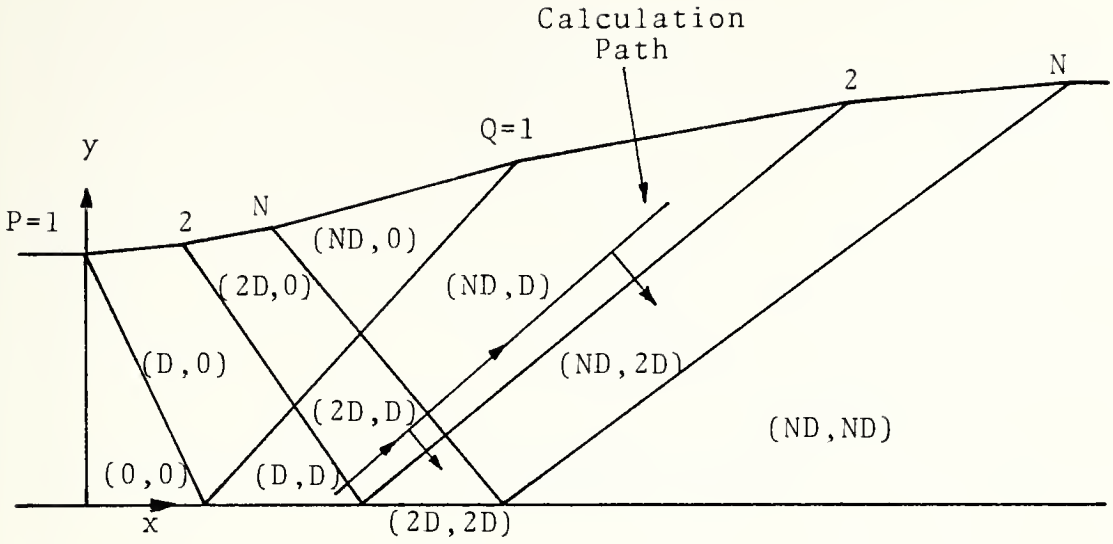
TABLE B-II. "NOZZLE" Program Listing (Continued)

```
NO. OF CHARACTERISTICS= 24
INPUT STEP SIZE= 0.045   INCHES
DESIRED MACH NO.= 1.4
TEST SECTION HEIGHT= 4.728   INCHES
P[1, 1    )= 0    P(2, 1    )= 2.120320636
P[1, 2    )= 0.044999760   P(2, 2    )= 2.120467686
P[1, 3    )= 0.089998799   P(2, 3    )= 2.120761783
P[1, 4    )= 0.134996636   P(2, 4    )= 2.121202926
P[1, 5    )= 0.179992792   P(2, 5    )= 2.121791108
P[1, 6    )= 0.224986786   P(2, 6    )= 2.122526324
P[1, 7    )= 0.269978137   P(2, 7    )= 2.123408566
P[1, 8    )= 0.314966364   P(2, 8    )= 2.124437825
P[1, 9    )= 0.359950988   P(2, 9    )= 2.125614089
P[1, 10   )= 0.404931528   P(2, 10   )= 2.126937346
P[1, 11   )= 0.449907504   P(2, 11   )= 2.128407582
P[1, 12   )= 0.494878436   P(2, 12   )= 2.130024782
P[1, 13   )= 0.539843842   P(2, 13   )= 2.131788927
P[1, 14   )= 0.584803244   P(2, 14   )= 2.133699999
P[1, 15   )= 0.629756161   P(2, 15   )= 2.135757978
P[1, 16   )= 0.674702112   P(2, 16   )= 2.137962841
P[1, 17   )= 0.719640619   P(2, 17   )= 2.140314566
P[1, 18   )= 0.764571201   P(2, 18   )= 2.142813126
P[1, 19   )= 0.809493379   P(2, 19   )= 2.145458496
P[1, 20   )= 0.854406672   P(2, 20   )= 2.148250647
P[1, 21   )= 0.899310601   P(2, 21   )= 2.151189550
P[1, 22   )= 0.944204687   P(2, 22   )= 2.154275172
P[1, 23   )= 0.989088449   P(2, 23   )= 2.157507481
P[1, 24   )= 1.03396141    P(2, 24   )= 2.160886443
```

TABLE B-III. Program "NOZZLE" Design Output

```
Q[1,1)= 1.099752663   Q(2,1)= 2.166056828
Q(1, 2    )= 1.795223097   Q(2, 2    )= 2.218426180
Q(1, 3    )= 2.084486737   Q(2, 3    )= 2.239257533
Q(1, 4    )= 2.317478495   Q(2, 4    )= 2.255271324
Q(1, 5    )= 2.521877660   Q(2, 5    )= 2.268640977
Q(1, 6    )= 2.708553266   Q(2, 6    )= 2.280254149
Q(1, 7    )= 2.883073968   Q(2, 7    )= 2.290531296
Q(1, 8    )= 3.048723217   Q(2, 8    )= 2.299742944
Q(1, 9    )= 3.207618128   Q(2, 9    )= 2.308058241
Q(1, 10   )= 3.36121201    Q(2, 10   )= 2.315592925
Q(1, 11   )= 3.510550085   Q(2, 11   )= 2.322429736
Q(1, 12   )= 3.656411756   Q(2, 12   )= 2.328629823
Q(1, 13   )= 3.799395299   Q(2, 13   )= 2.334239551
Q(1, 14   )= 3.93997101    Q(2, 14   )= 2.339294793
Q(1, 15   )= 4.078516025   Q(2, 15   )= 2.343823742
Q(1, 16   )= 4.215337948   Q(2, 16   )= 2.347848829
Q(1, 17   )= 4.350691379   Q(2, 17   )= 2.351388069
Q(1, 18   )= 4.484789768   Q(2, 18   )= 2.354456026
Q(1, 19   )= 4.6178141  Q(2, 19    )= 2.357064520
Q(1, 20   )= 4.749919395   Q(2, 20   )= 2.359223162
Q(1, 21   )= 4.881239644   Q(2, 21   )= 2.360939759
Q(1, 22   )= 5.011891617   Q(2, 22   )= 2.362220623
Q(1, 23   )= 5.141977849   Q(2, 23   )= 2.363070819
Q(1, 24   )= 5.271588992   Q(2, 24   )= 2.36349436
A/A*= 1.114687241
PERCENT ERROR IN A/A*= 0.021389147
```

TABLE B-III. Program "NOZZLE" Design Output (Continued)



N = number of characteristics

U = total turning angle

D = U/2N

P(1,N) = x coordinates of P = 1 to N

P(2,N) = y coordinates of P = 1 to N

Q(1,N) = x coordinates of Q = 1 to N

Q(2,N) = y coordinates of Q = 1 to N

R(3,N) = angle of left running characteristic for each step

P(3,N) = angle of right running characteristics for each step

Figure B-1. Notation for Program "NOZZLE"

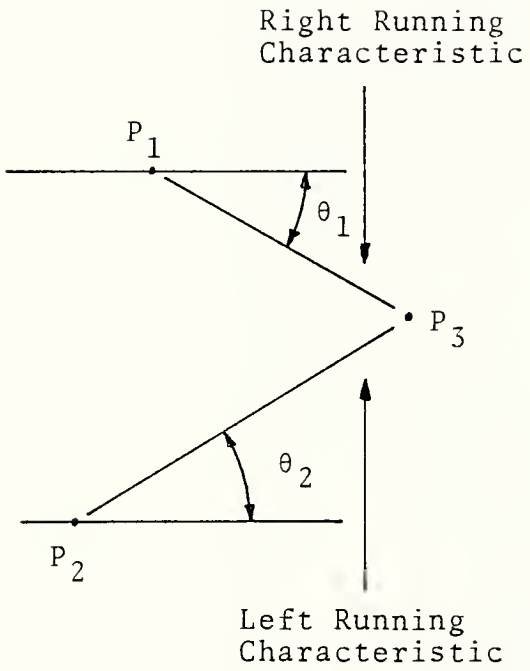


Figure B-2. Point to Point Calculation Illustration for Program "NOZZLE".

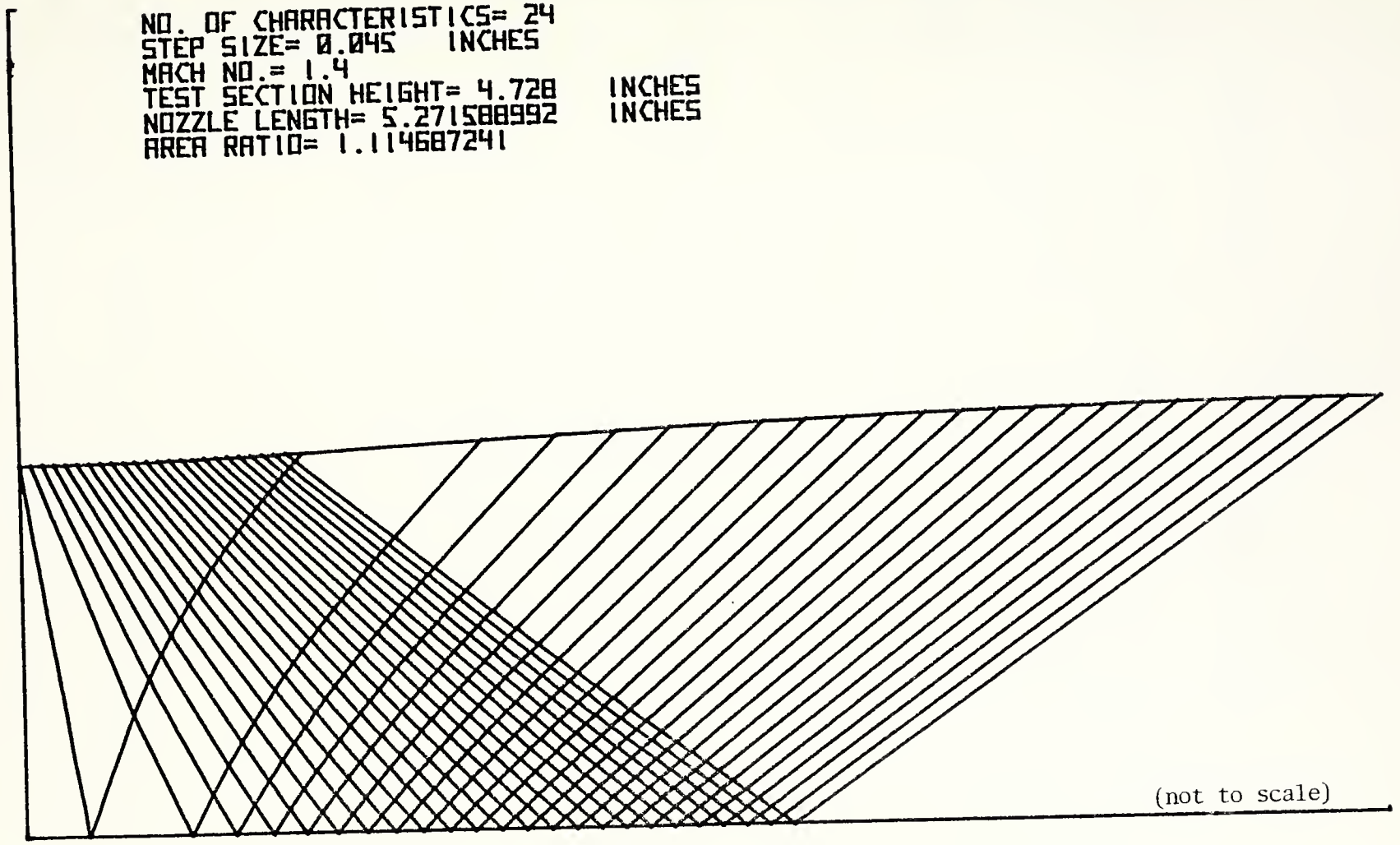


Figure B-3. Program "NOZZLE" Characteristics Plot

APPENDIX C

PRESSURE DROP IN A SUDDEN EXPANSION PROCESS

Figure C-1 shows an expansion process in a duct, from uniform conditions at station 1 to uniform conditions downstream at a larger area at station 2. The assumptions are:

(1) The flow is steady and adiabatic with perfect gas behavior, and

(2) Gravitational forces are negligible.

From continuity

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m} \tag{C-1}$$

From momentum conservation

$$-\rho_1 A_1 V_1^2 + \rho_2 A_2 V_2^2 = P_1 A_1 - P_2 A_2 \tag{C-2}$$

From energy conservation

$H_{t1} = H_{t2}$ (where here, H denotes enthalpy)

which for a perfect gas gives

$$T_{t1} = T_{t2} \tag{C-3}$$

If

$$V_o = \sqrt{2 c_p T_t}$$

is the "limiting" velocity, a non-dimensional velocity, X, can be defined as

$$X = \frac{V}{V_o}$$

From the perfect gas equation of state

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \quad \text{and} \quad \frac{P_{t1}}{\rho T_{t1}} = \frac{P_{t2}}{\rho T_{t2}} \tag{C-4}$$

Since the flow is adiabatic

$$T_{t1} = T_{t2} = T_t$$

and Eq. C-1 then becomes

$$\frac{\rho_1}{\rho_2} = \frac{X_2}{X_1} \frac{A_2}{A_1} \tag{C-5}$$

also

$$T_t = T + \frac{V^2}{2c_p} = T + X^2 T_t$$

so that

$$T_t(1-X_1^2) = T_1$$

and

$$T_t(1-X_2^2) = T_2 \tag{C-6}$$

From Eq. C-4, Eq. C-5, and Eq. C-6

$$\frac{P_1}{P_2} = \frac{(1-X_1^2)}{(1-X_2^2)} \frac{X_2}{X_1} \frac{A_2}{A_1} \tag{C-7}$$

Dividing Eq. C-2 by $p_1 A_1 V_1$ and using Eq. C-5 and Eq. C-7

$$\left[\frac{X_2}{X_1} - 1\right] = \frac{P_1}{\rho_1 V^2} \left(\frac{A_2}{A_1}\right)^2 \left[1 - \frac{(1-X_2^2)}{(1-X_1^2)} \frac{X_1}{X_2} \frac{A_1}{A_2} \right] \tag{C-8}$$

and since

$$\frac{P_1}{\rho_1 V_1^2} = \frac{RT_1 P_1}{P_1 V_1^2} = \frac{k-1}{2k} \frac{2c_p T_t (1-X_1^2)}{V_1^2} = \frac{k-1}{2k} \left(\frac{1-X_1^2}{X_1^2}\right)$$

Eq. C-8 becomes

$$\left[\frac{k+1}{k-1} X_1^2\right] \left(\frac{X_2}{X_1}\right)^2 - \left[\frac{A_2}{A_1} (1 - X_1^2) + \frac{2k}{k-1} X_1^2\right]\left(\frac{X_2}{X_1}\right) + 1 = 0 \quad \text{(C-9)}$$

P_2 is obtained by solving Eq. C-9 for X_2 and substituting for X_2 in Eq. C-7.

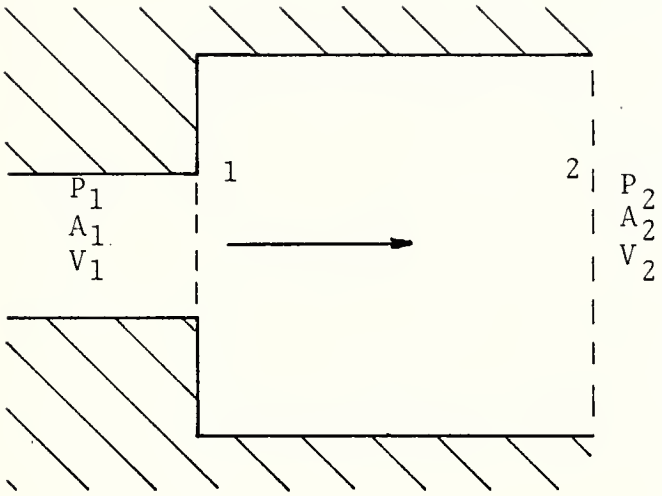
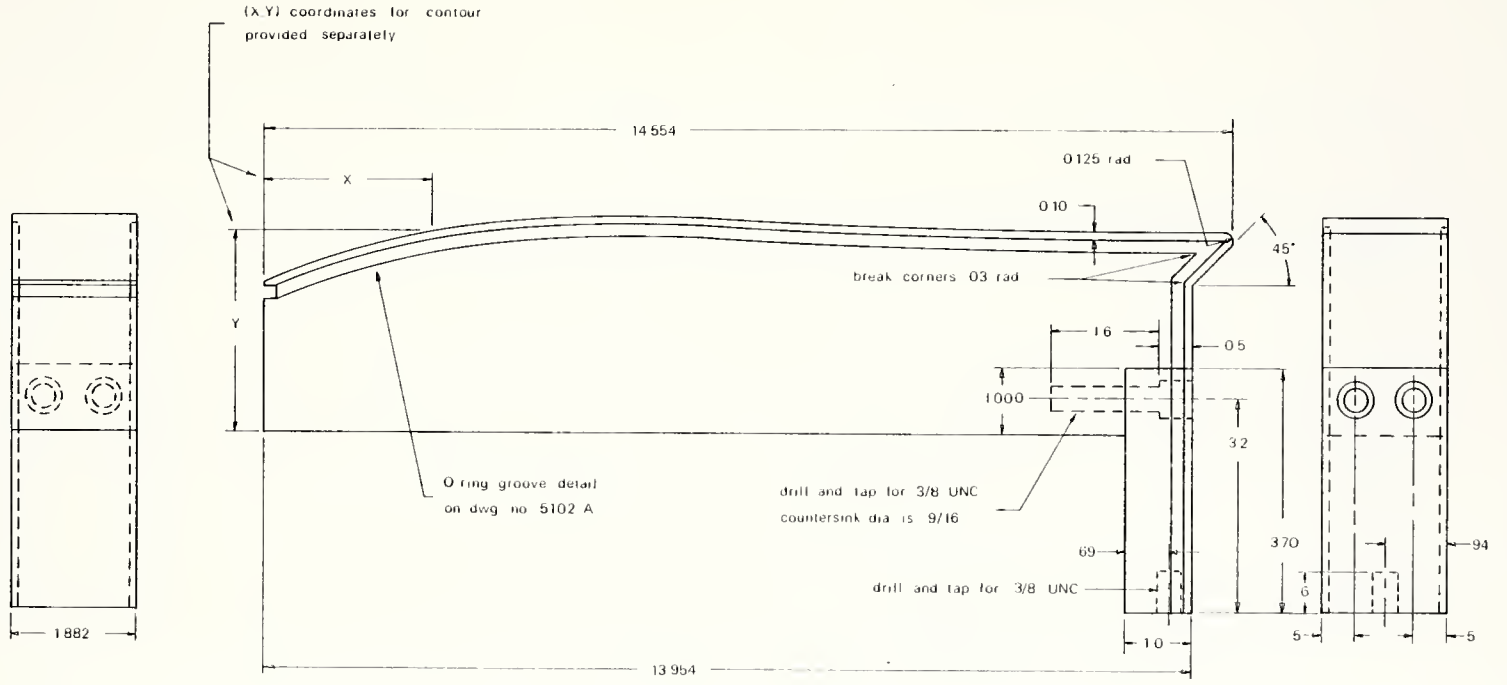


Figure C-1. Expansion Process in a Duct

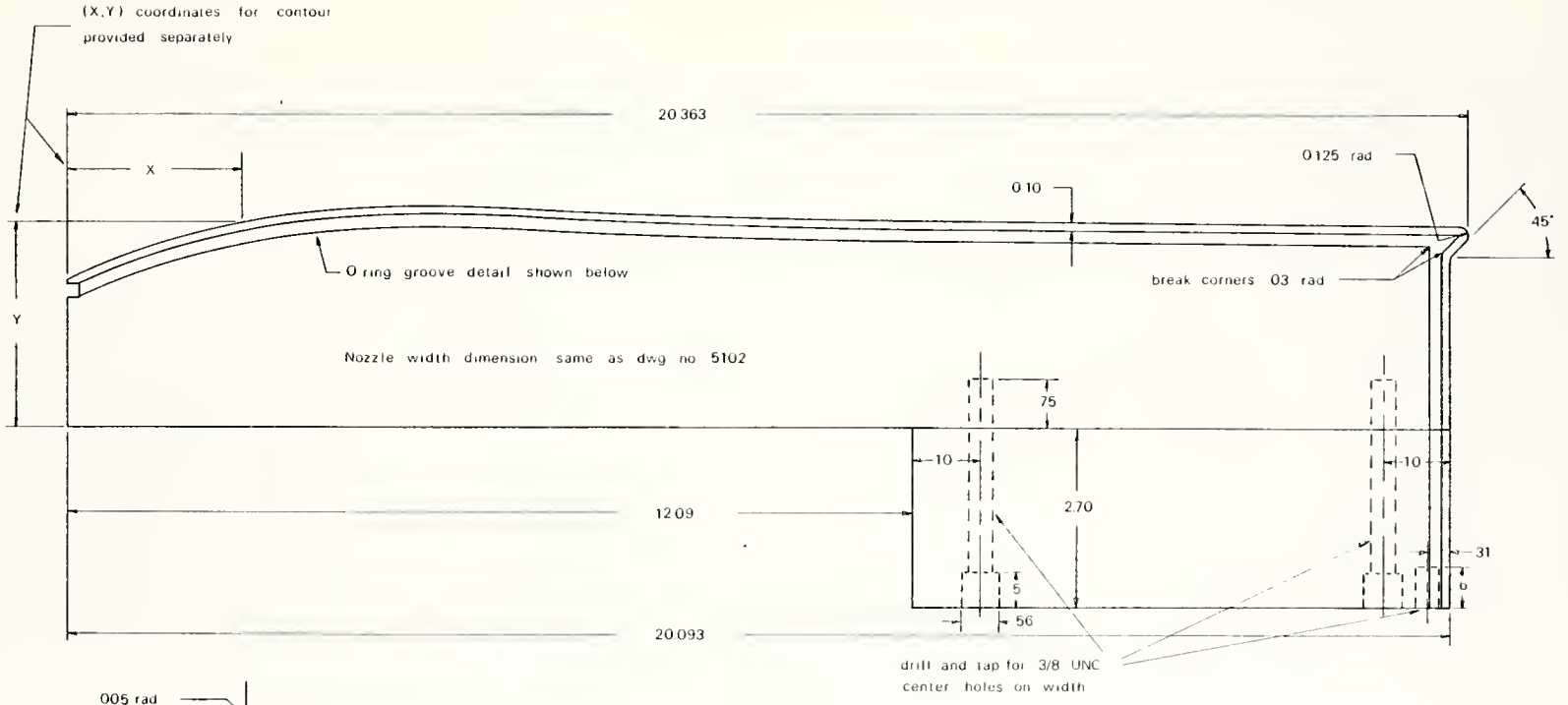
APPENDIX D

MACHINE DRAWINGS OF COMPONENTS

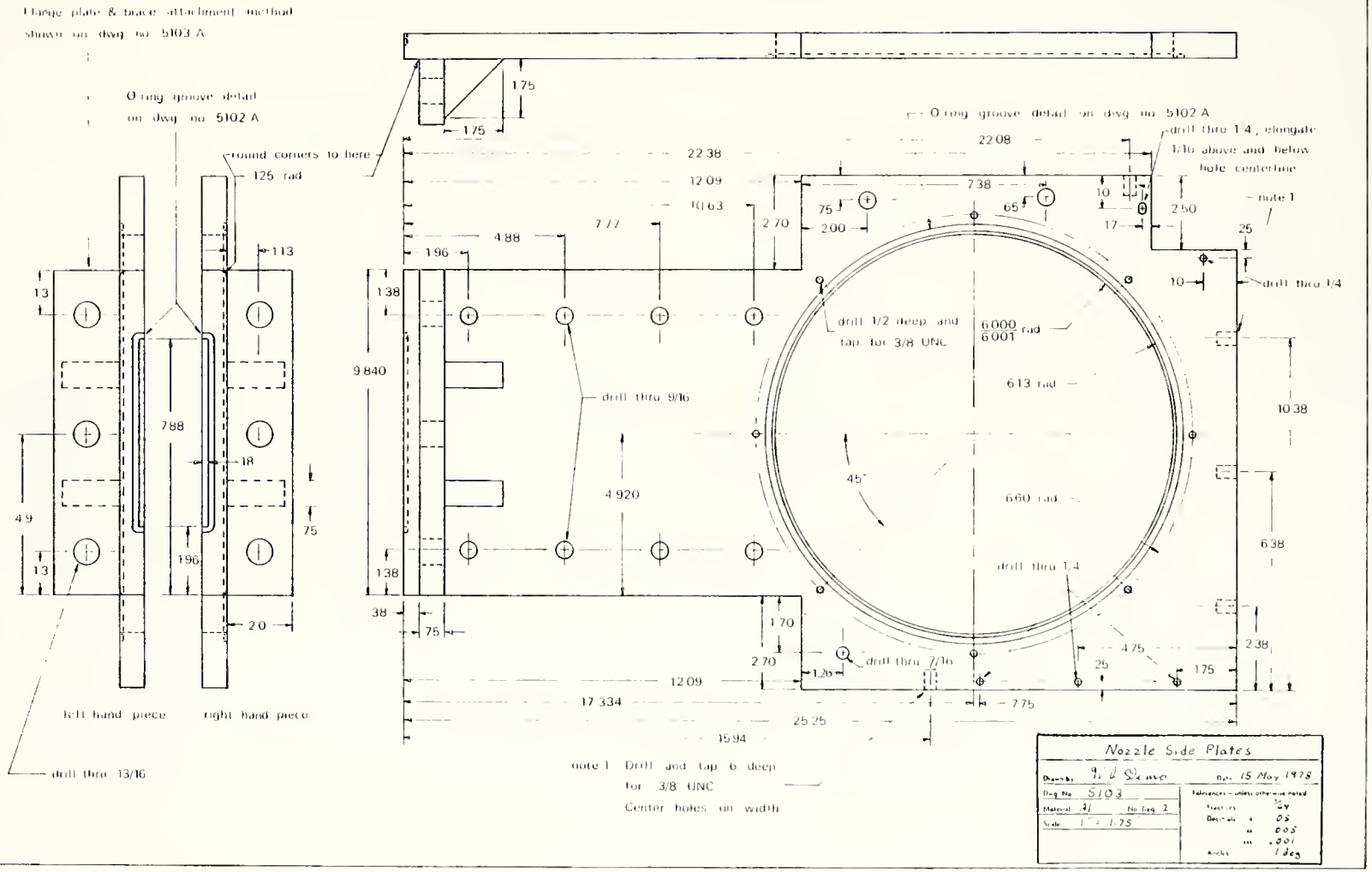
Drawing No.	Title
5102	Lower Nozzle Block
5102-A	Upper Nozzle Block
5103	Nozzle Side Plates
5103-A	Nozzle Side Plate Flange Mounting
5104	Diffuser End Plates
5105	Window and Window Support Ring
5106	End Passage Exhaust Flanges
5107	Main Exhaust Flange
5108	Blades and Mounting Pins
5109	Window Blade Mounting Holes
5110	Passage Exhaust Scoops

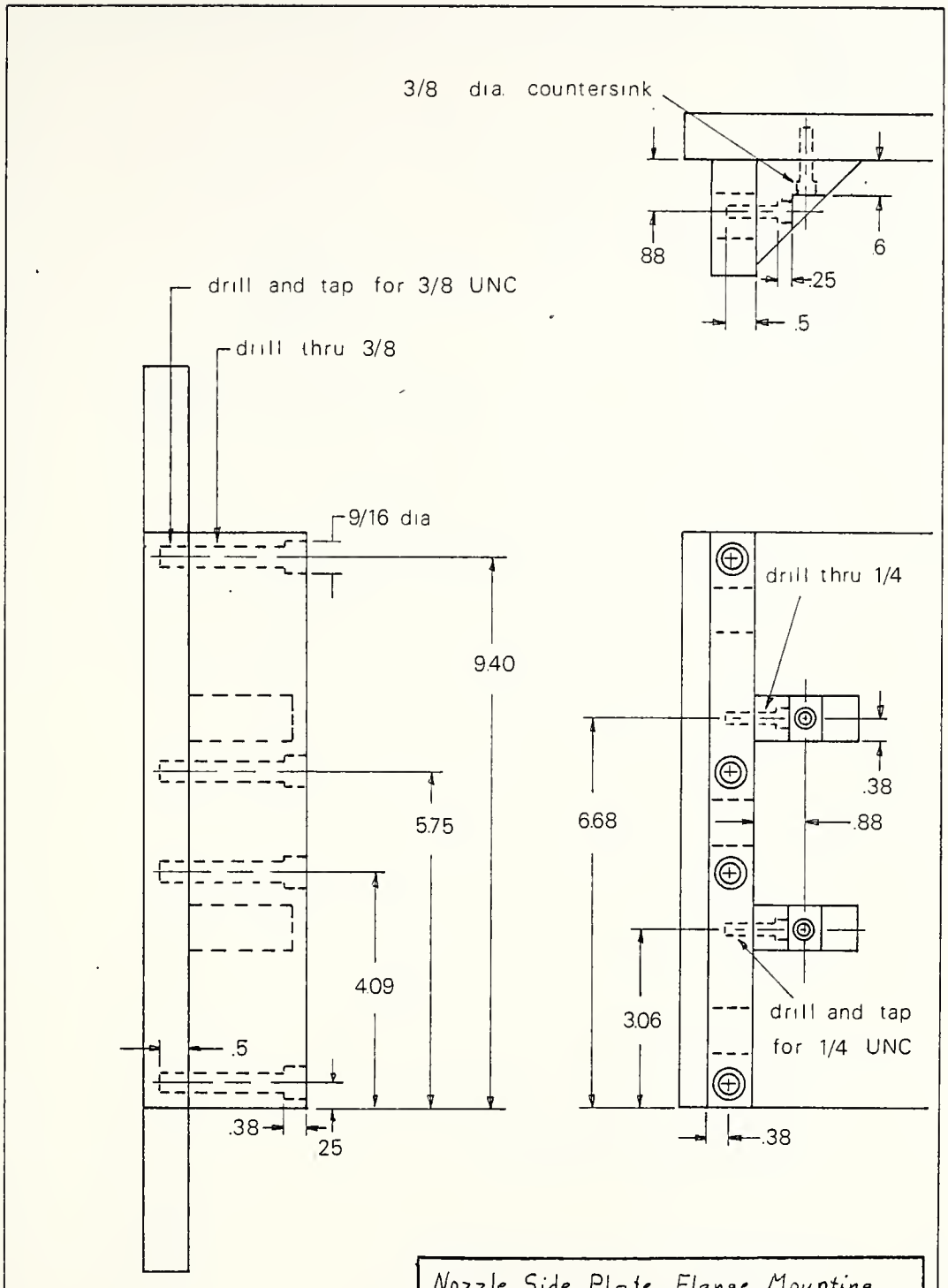


Lower Nozzle Block		
Drawn by W J Szmo		Date 9 May 1978
Dwg No 5102		Tolerances – unless otherwise noted.
Material Al	No. Req. 1	Fractions ± 1/64
Scale Full		Decimals .x ± .05
		.xx ± .025
		.xxx ± .001
		Angles ± 1 deg



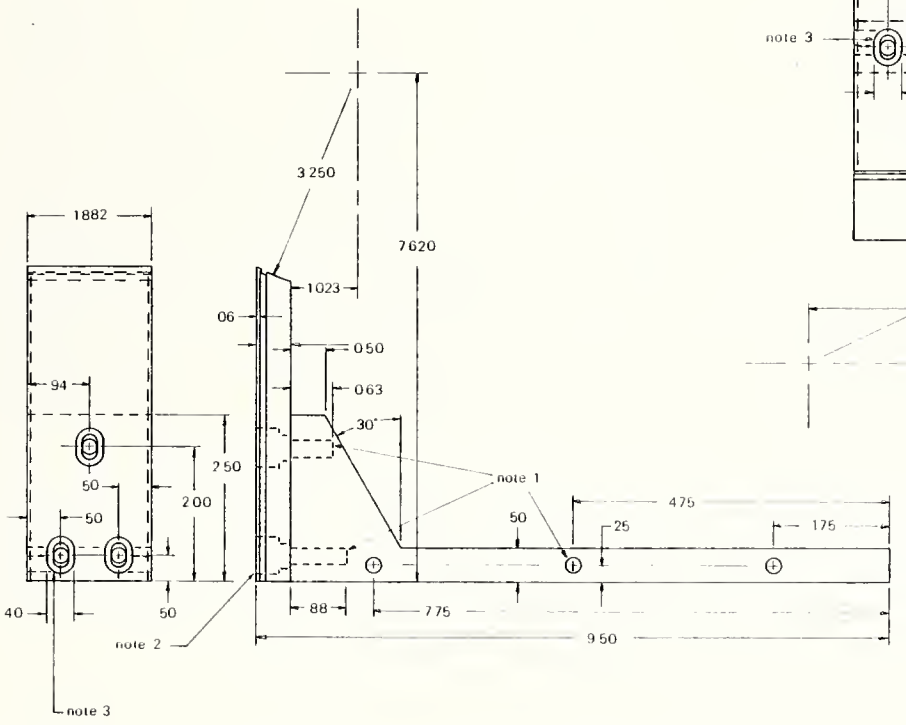
Upper Nozzle Block		
Drawn by: W.B. Steme		Date: 9 May 1978
Dwg No: 5102 - A		Tolerances — unless otherwise noted
Material: Al	No. Req: 1	Fractions: 1/64
Scale: Full		Decimals: x .05, xx .005, xxx .001
		Angles: 1 deg

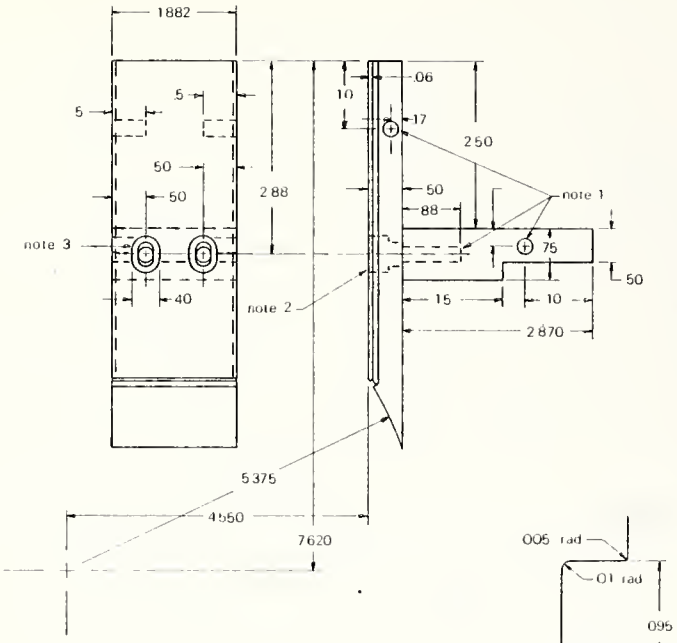

Flange plate & brace attachment method
Shown on dwg no 5103 A
O ring groove detail
on dwg no 5102 A
Round corners to here
125 rad
113
13
788
18
49
75
196
13
20
left hand piece
right hand piece
drill thru 13/16
175
175
2238
2209
1209
1063
777
488
196
270
738
75
65
200
10
17
250
25
note 1
drill thru 1/4
drill thru 1/4, elongate
1/16 above and below
hole centerline
O ring groove detail on dwg no 5102 A
drill 1/2 deep and
tap for 3/8 UNC
6000/6001 rad
613 rad
660 rad
45°
drill thru 9/16
9840
4920
138
138
38
75
1038
638
238
drill thru 1/4
drill thru 7/16
170
270
126
1209
17334
2525
1594
775
475
25
175
note 1 Drill and tap 6 deep
for 3/8 UNC
Center holes on width
Nozzle Side Plates
Drawn by W. A. Stamm
Date 15 May 1978
Dwg No 5103
Material 31
No Req 2
Scale 1" = 1.75
Tolerances - unless otherwise noted
Fractions ±1/64
Decimals x .05
.xx .005
.xxx .001
Angles 1 deg

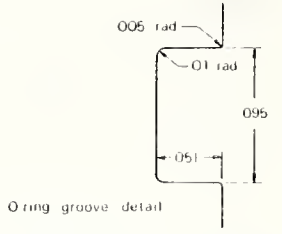


Nozzle Side Plate Flange Mounting		
Drawn by: W.A. Demo	Date 12 June 78	
Dwg. No. 5103-A	Tolerances – unless otherwise noted:	
Material A1 No. Req.	Fractions ± 1/64	
Scale 1" = 1.75"	Decimals .x ± .05	
	.xx ± .005	
	.xxx ± .001	
	Angles ± 1 deg.	

notes 1 Drill and tap all holes for 1/4 UNC
2 Countersink holes 5/16
3 Hole elongations are 1/16 above and below hole centerline

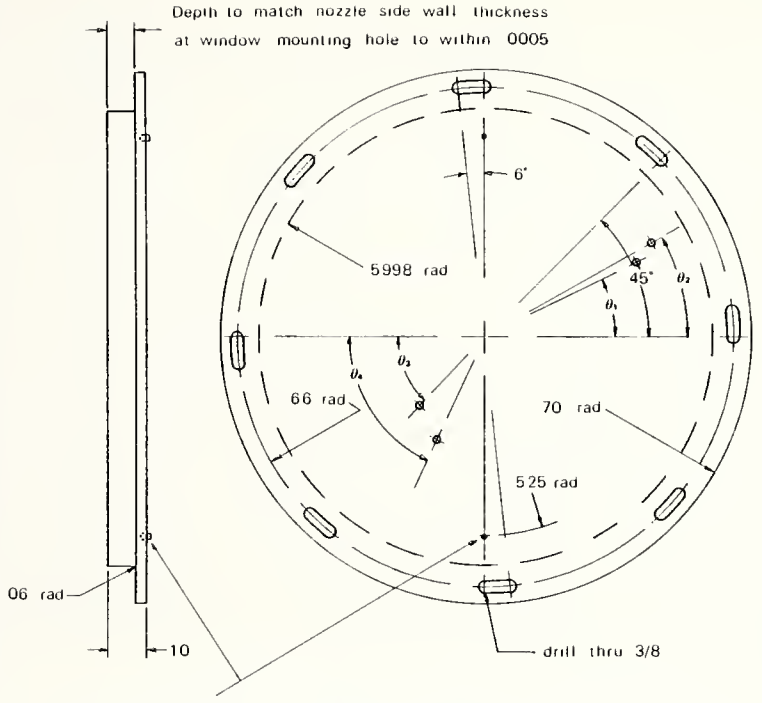


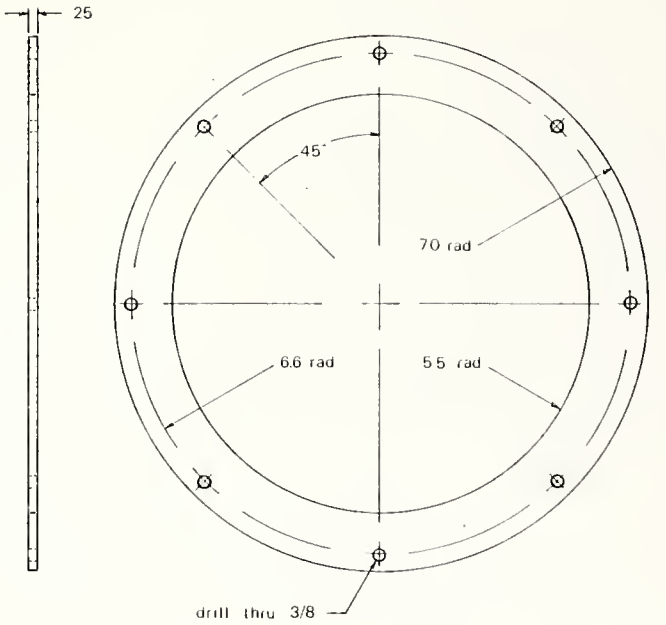




O ring groove detail

Diffuser End Plates	
Drawn by W B Stone	Date 1 June 1978
Dwg No 5104	Tolerances - unless otherwise noted
Material 41 No Req 10a	Fractions 1/64
Scale Full	Decimals .xx .05 .xxx .005 .xxxx .001
	Angles 1°





Press in 1/8 dia pins to be used for axis location

Window - Plexiglass
2 reqd - one as above and the other of opposite hand

θ_1 - 25° 38' r1 - 4478
θ_2 - 28° 57' r2 - 5036
θ_3 - 46° 32' r3 - 2578
θ_4 - 64° 36' r4 - 2968

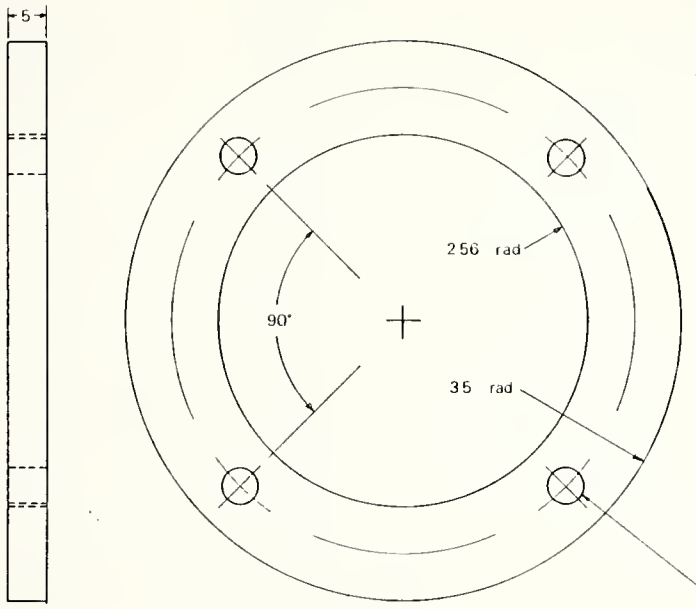
drill holes thru 309

Window Support Ring
2 reqd Aluminum

Window and Window Support Ring

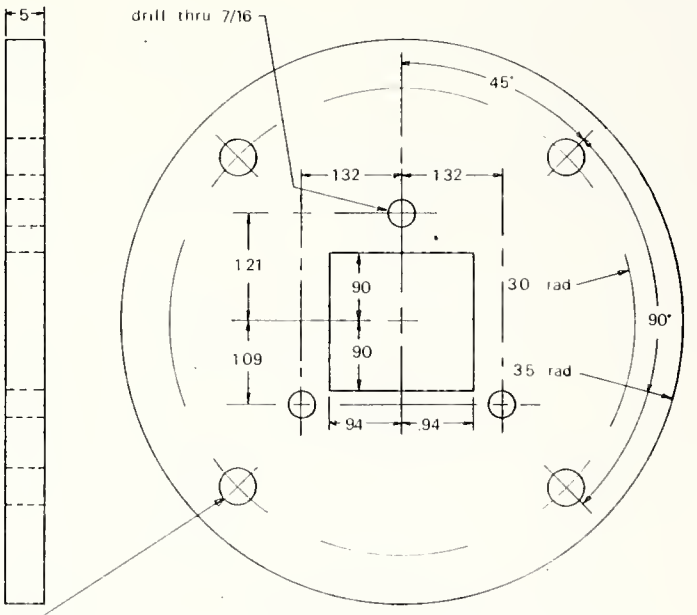
Drawn by: W P Stone — Date: 7 June 1978
Dwg. No. 5105
Material as noted — unless otherwise noted
Scale one-half

Tolerances — unless otherwise noted.
Fractions: ± 1/64
Decimals: x = .05, .xx = .005, .xxx = .001
Angles: ± 1 min



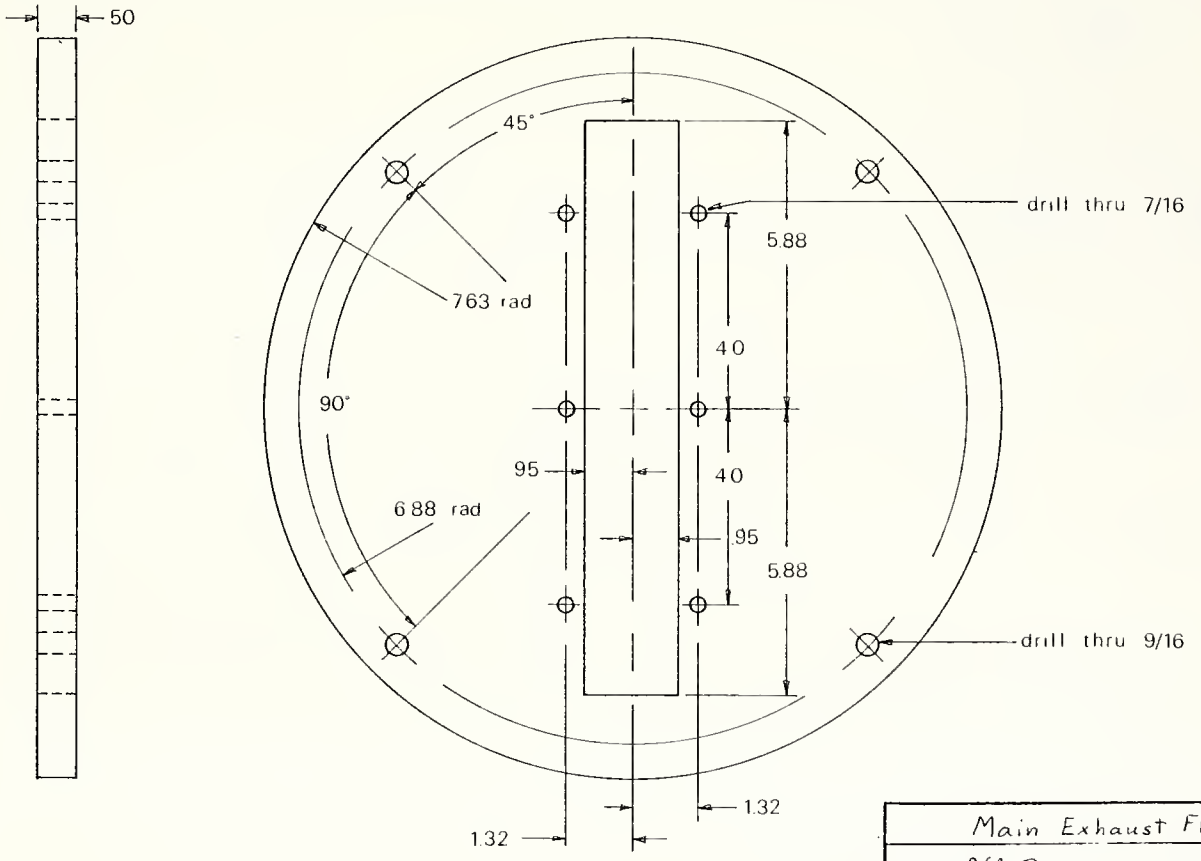
Plexiglass Flange

drill thru 9/16

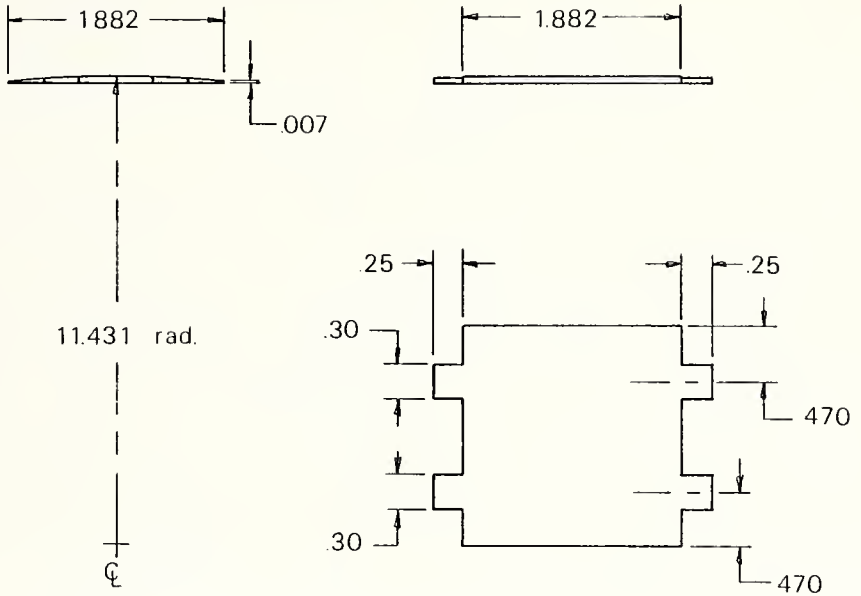


Aluminum Flange

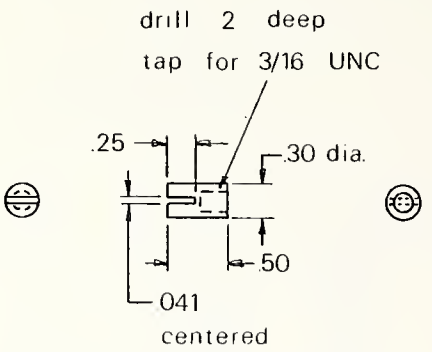
End Passage Exhaust Flanges		
Drawn by: W. P. Demo	Date 1 June 1978	Tolerances – unless otherwise noted
Dwg No 5106		fractions ± 1/64
Material as noted No Req 2 ea		Decimals .x ± .05
Scale full		.xx ± .025
		.xxx ± .001
		Angles ± 1deg



Main Exhaust Flange		
Drawn by: W.J. Demo		Date 2 June 1978
Dwg. No. 5107		Tolerances – unless otherwise noted:
Material Al	No. Req. 1	Fractions ± 1/64
Scale one-half		Decimals .x ± .05
		.xx ± .005
		.xxx ± .001
		Angles ± 1deg.

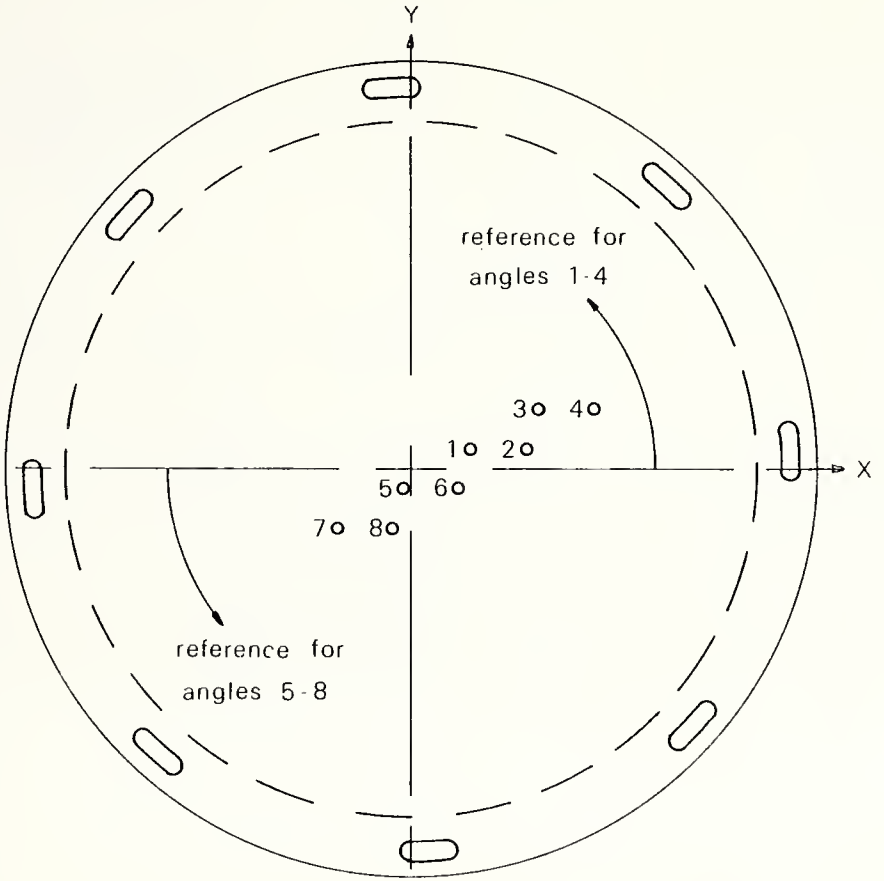


Blades - 7075-T6 Al
4 required

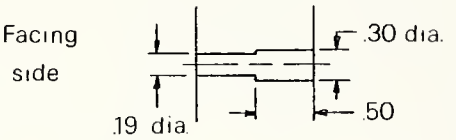


Mounting Pins - Al. stock
8 required

Blades and Mounting Pins		
Drawn by: W. J. Demo	Date 8 June 1978	
Dwg. No. 5108	Tolerances – unless otherwise noted:	
Material Al 7075-T6 No. Req. as noted	Fractions ± 1/64	
Scale full	Decimals .x ± .05	
	.xx ± .005	
	.xxx ± .001	
	Angles ± ———	



Hole Detail



Hole#	X(in.)	Y(in.)	R(in.)	Theta(deg.)
1				
1	1.051	0.358	1.111	18.80
2	1.992	0.358	2.024	10.19
3	2.213	1.033	2.443	25.03
4	3.154	1.033	3.319	18.14
5	-0.110	-0.317	0.336	70.82
6	0.831	-0.317	0.889	159.08
7	-1.272	-0.993	1.614	37.97
8	-0.331	-0.993	1.047	71.55

Window Blade Mounting Holes

Drawn by: W.F. Demo Date 10 June 1978

Dwg. No. 5109

Material — No. Req. —

Scale one-half

Tolerances – unless otherwise noted:

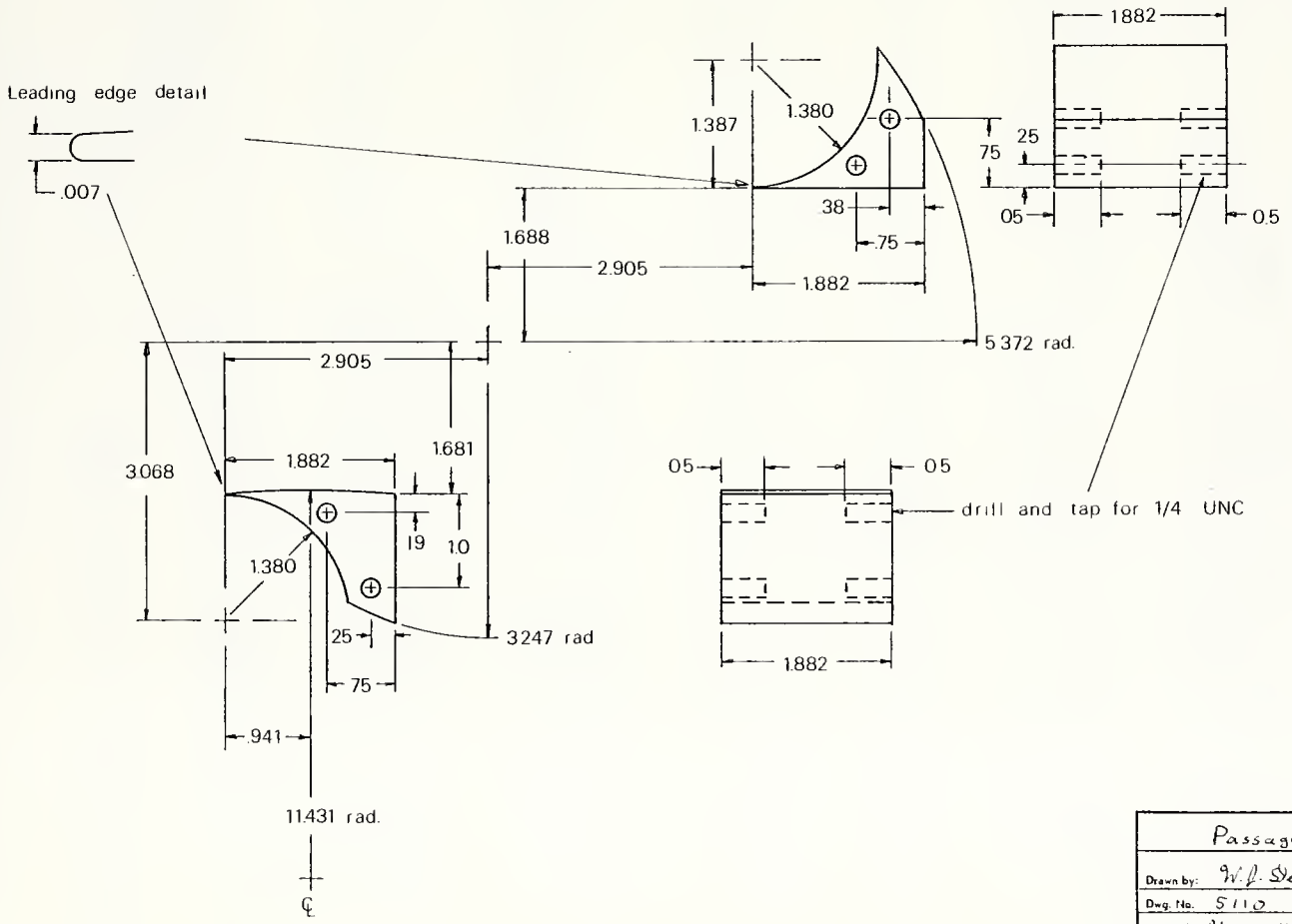
Fractions ± 1/64

Decimals .x ± .05

.xx ± .025

.xxx ± .001

Angles ± 1 min.



Passage Exhaust Scoops	
Drawn by: W.F. Stemo	Date 7 June 1978
Dwg. No. 510	Tolerances – unless otherwise noted:
Material Al No. Req. ea	Fraction: ± 1/2
Scale full	Decimals .x ± .05
	.xx ± .025
	.xxx ± .021
	Angles ± 1deg.

LIST OF REFERENCES

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2. Shreeve, R. P., Probes and Measurement Techniques from Tests of a Small Transonic Axial Compressor, AIAA Paper 78-969, to be presented at the AIAA/SAE 14th Propulsion Conference, July 25-27, 1978, Las Vegas, Nevada.

3. Shreeve, R. P., Dodge, F. J., and Hawkins, W. R., Probe Measurements of Velocity and Losses from a Small Axial Transonic Rotor, AIAA Paper 78-1198, to be presented at the AIAA 11th Fluid and Plasma Dynamics Conference, July 10-12, 1978, Seattle, Washington.

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12. Ruptash, J., Growth of Boundary Layer in Supersonic Nozzles, National Aeronautical Establishment.

13. National Aeronautics and Space Administration Special Report NASA SP-36, Aerodynamic Design of Axial Flow Compressors (NASA SP-36), Edited by Irving A. Johnson and Robert O. Bullock, 1965.

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